21/21/2006

\[ \begin{bmatrix}
1 & 0 & -\frac{2}{3} & \frac{1}{3} \\
0 & 1 & \frac{2}{3} & -\frac{1}{3} \\
0 & 0 & 0 & 0
\end{bmatrix} \]

\[ x - \frac{2}{3} = \frac{1}{3} \]

\[ x = \frac{2}{3} z + \frac{1}{3} \]

\[ y + \frac{1}{3} z = -\frac{1}{3} \]

\[ y = -\frac{1}{3} - \frac{1}{3} z \]

\[ \frac{2}{3} z + \frac{1}{3} z = -\frac{1}{3} - \frac{1}{3} z, \ z = 2 \]
Identity matrix

\[
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
\]

Arithmetic

\[5 \cdot 1 = 5 \cdot 5 = 5\]

1 is the unit identity element.

\[X \cdot 1 = X \cdot X = X\]

Matrices

\[A \cdot I = I \cdot A = A\]

where \(A\) is a square matrix.

\[
\begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\cdot
\begin{bmatrix}
1 & 0 \\
0 & 1
\end{bmatrix}
= \begin{bmatrix}
1 & 2 \\
3 & 4
\end{bmatrix}
\]

\[A \cdot I = A\]
In arithmetic
\[ S \cdot \left( \frac{1}{S} \right) = 1 \]
\[ \frac{1}{S} \text{ is the multiplicative inverse of } S \]
\[ X \cdot \frac{1}{X} = 1 \]

\[ A \cdot A^{-1} = A^{-1} \cdot A = I \]

\[ A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

Find \( A^{-1} \)

Calculating \( A^{-1} \):
\[ [A]^{-1} = \begin{bmatrix} -\frac{2}{13} & \frac{1}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix} \]

Find inverse by 

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Using 

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

Convert \( A \) into \( I \) and 

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

will become multiplied into \( A^{-1} \)

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \]

\[ \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \]

\[ \begin{bmatrix} -\frac{2}{13} & \frac{1}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix} \]

\[ A^{-1} = \begin{bmatrix} -\frac{2}{13} & \frac{1}{13} \\ \frac{3}{13} & -\frac{1}{13} \end{bmatrix} \]
Find $A^{-1}$ for $A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 4 & 5 \\ 2 & 1 & 4 \end{bmatrix}$

Do on the calculator

$A^{-1} = \begin{bmatrix} -19/21 & 4/21 & 2/3 \\ 2/21 & 4/21 & -1/3 \\ 3/7 & -1/7 & 0 \end{bmatrix}$
3.4

Write the matrix equation for this system.

\[
\begin{align*}
3x - 4y &= 11 \\
2x + 3y &= -4
\end{align*}
\]

\[
\begin{pmatrix}
3 & -4 \\
2 & 3
\end{pmatrix}
\begin{pmatrix}
x \\
y
\end{pmatrix} =
\begin{pmatrix}
11 \\
-4
\end{pmatrix}
\]

Solve this system by using the inverse of a matrix.

\[
A \cdot X = B
\]

\[
A^{-1} \cdot A \cdot X = A^{-1} \cdot B
\]

mutl on the left by \( A^{-1} \)

\[
I \cdot X = A^{-1} \cdot B
\]

\[
X = A^{-1} \cdot B
\]

\[
\begin{pmatrix}
A^{-1} \cdot B
\end{pmatrix}
\]

\[
\begin{pmatrix}
[-4] \\
[1] \\
[-2]
\end{pmatrix}
\]
3.5

Leontief Input-Output Model

1973 Nobel Prize

Economy into 500 sectors
42 sectors later

Gross Production Matrix $X$

$$X = \begin{bmatrix}
    x_1 \\
    x_2 \\
    x_3 \\
    \vdots \\
    x_m
\end{bmatrix}$$

Technology matrix $A$ (Leontief matrix)

$A$ shows the relationships between the different sectors of the economy.
For each 1,000 units of RM produced, how many units of RM, 0.008 units of ST, and 0.008 units of FT are required?

To make 1 unit of raw material, 0.008 units of RM, 0.008 units of FT, and 0.008 units of SL are produced.

In input, 0.008 units of RM, 0.008 units of FT, and 0.008 units of SL are used to make 1 unit of raw material.
Open Leontief Model - not all sectors of the economy are included.

Surplus or Final Demands will exist for consumers to purchase.

\[ D \]

(Gross Product) - (what is consumed in the production process) = (Final Demands (Surplus))

\[ X - AX = D \]

\[ I \cdot X - AX = D \]

\[ (I - A) \cdot X = D \]

\[ (I - A)^{-1} (I - A) \cdot X = (I - A)^{-1} \cdot D \]

\[ I \cdot X = (I - A)^{-1} \cdot D \]

\[ X = (I - A)^{-1} \cdot D \]
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

Surplus
60 ag.
70 min
desired

Find \( X \)

\[
X - AX = D
\]

\[
I - A \cdot X = D
\]

\[
(I - A)^{-1} \cdot X = D
\]

\[
X = (I - A)^{-1} \cdot D
\]

\[
D = \begin{bmatrix} 60 \\ 70 \end{bmatrix}
\]

\[
X = \begin{bmatrix} 100 \\ 100 \end{bmatrix}
\]

Watch \( (I - A)^{-1} \) around \( I \) ag.
100 units Min to reach demand desired,
Thursday
3, 4  Do these assigned
3, 5  Read

Do 3, 5  1-10 odd
13, 15, 17

March 2 Thursday Text 2