11/9/2005

T1  T2  T3  T4  + Final

Last day for W is Friday, Nov 11
3.8 Newton's Method for Approximating Zeros of a Function

Ex. \( f(x) = 2x^2 - 3 \)

Find a zero of the function or \( x \)-intercept of the graph

\[
0 = 2x^2 - 3
\]

\[
3 = 2x^2
\]

\[
\frac{3}{2} = x^2
\]

\[
x = \pm \sqrt{\frac{3}{2}}
\]

\[
x \approx \pm 1.224744871
\]
Newton's Method

Strategy

Find x-intercept zero of f(x)

1st guess call it x. The x-intercept of this tangent line at \((x_i, f(x_i))\) will give us our new "guess" (approximation)

Call it \(x_2\)
By the Intermediate Value Theorem, if \( f \) takes on all values between \( f(a) \) and \( f(b) \), there exists a \( c \in (a, b) \) such that \( f(c) = 0 \).

The equation of the tangent line at \((x_1, f(x_1))\) is:

\[
m = f'(x_1) \quad \text{and} \quad y - f(x_1) = m(x - x_1)
\]

When \( y = 0 \), the tangent line intersects the x-axis.

\[
O - f(x_1) = f'(x_1)x - f(x_1)x
\]

So we find:

\[
f(x_1) - f(x_1) = f'(x_1)x
\]

\[
x = \frac{f(x_1)}{f'(x_1)}
\]

The new approximation will be the next of \( x \):

\[
x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}
\]

Generalize:

\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]
\[ f(x) = 2x^2 - 3 \]

Let \( x_n = 1 \) \( f(x) = 4x \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( x_n )</th>
<th>( f(x_n) )</th>
<th>( f'(x_n) )</th>
<th>( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>-1</td>
<td>8</td>
<td>( x_2 = 1 - \frac{1}{8} = \frac{7}{8} )</td>
</tr>
<tr>
<td>2</td>
<td>( \frac{5}{4} )</td>
<td>( \frac{3}{4} )</td>
<td>( 1.25 )</td>
<td>( x_3 = \frac{5}{4} - \frac{\frac{3}{4}}{1.25} = 1.225 )</td>
</tr>
<tr>
<td>3</td>
<td>1.225</td>
<td>( \frac{0.0125}{1.25} )</td>
<td>( 4.9 )</td>
<td>( x_4 = 1.225 - \frac{\frac{0.0125}{1.25}}{4.9} = 1.2247 )</td>
</tr>
<tr>
<td>4</td>
<td>1.2247</td>
<td></td>
<td></td>
<td>( x_5 = 1.2247 )</td>
</tr>
</tbody>
</table>

Title: Nov 9 - 9:06 AM (5 of 8)
3.9 Differentials

Derivatives

\[ y = f(x) \]

\[ y' = f'(x) = \frac{dy}{dx} \]

This was not a quotient.

Now we talk about

\[ dy \text{ is differential of } y \]

\[ dx \text{ is differential of } x \]

Now \( \frac{dy}{dx} \) is a quotient of 2 differentials.
dy is the change in y that occurs as you move along the curve dx units.

\[ \Delta y = f(x+dx) - f(x) \]

\[ \frac{dy}{dx} \] \quad \text{change in y divided by the change in x as you move on}

The tangent line

dy is the change in y that occurs as you move dx units along the tangent line to the curve at P.

The differential of x is dx and
\[ dx = \Delta x \]

\[ dy \] \quad \text{is the differential of y}

\[ \frac{dy}{dx} = f'(x) \]

\[ dy = f'(x) \, dx \]
Friday

Review any missed test

Do 3.8 from assigned

3.9 Read carefully

3.9 7

Next test Friday Dec 9

Last day of class Mon Dec 12

Final Exam Friday Dec 16

10:15 - 12:15

PH 112