Theorem 3.6: First Derivative Test for Extrema

Let $c$ be a critical number of the function $f$. If $f$ is continuous in some open interval containing $c$, and $f$ is differentiable on that interval except possibly at $x = c$, then

1. If $f'(c) = 0$ and $f''(c) > 0$, then $f(c)$ is a local minimum.
2. If $f'(c) = 0$ and $f''(c) < 0$, then $f(c)$ is a local maximum.
3. If $f'(c) = 0$ and $f''(c) = 0$, then the test is inconclusive.

No Extrema points.
$$f(x) = (x+2)^2 (x-1)$$

$$f'(x) = (x+2)^2 + (x-1) \cdot 2(x+2)' \cdot (1)$$

$$f'(x) = (x+2)^2 - 2(x-1)(x+2)$$

$$f'(x) = (x+2)[(x+2) + 2(x-1)]$$

$$f'(x) = (x+2)[x+2 + 2x-2]$$

$$f'(x) = 3x(x+2)$$

$$g = 3x(x+2)$$

$$x = 0 \quad x+2 = 0 \quad x = -2$$

<table>
<thead>
<tr>
<th>$x-2$</th>
<th>$x = -2$</th>
<th>$x &lt; x_0$</th>
<th>$x = 0$</th>
<th>$x &gt; 0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x - 3$</td>
<td>$x - 1$</td>
<td>$x = 1$</td>
<td>$x = -1$</td>
<td>$x = 1$</td>
</tr>
<tr>
<td>$f$</td>
<td>$0$</td>
<td>$-1$</td>
<td>$0$</td>
<td>$4$</td>
</tr>
<tr>
<td>$f'$</td>
<td>$+$</td>
<td>$-$</td>
<td>$0$</td>
<td>$+$</td>
</tr>
<tr>
<td>Conclusion</td>
<td>INC</td>
<td>Max</td>
<td>INC</td>
<td>DEC</td>
</tr>
</tbody>
</table>
\[
\frac{d^2 y}{dx^2} = \frac{d}{dx} \left( \frac{dy}{dx} \right)
\]

This gives us the instantaneous rate of change of the 1st derivative with respect to x.

If the 2nd derivative is +, that means the 1st derivative is increasing.

If the 2nd derivative is -, that means the 1st derivative is decreasing.
Concave Downward

$f''$ is Negative

Spill water
tangent lines are
above the graph

$f''$ is Decreasing

Concave Upward

$f''$ is Positive

Tangent lines are
below the graph

$f'(x)$ is Increasing

Point of Inflection

Concave Up

Concave Down

Holds water
The 3,7 Tests for concavity

Def Point of Inflection - a point on the graph where the concavity changes. Possible points of inflection occur where $f''(c) = 0$ or $f''(c)$ DNE but $f(c)$ does exist.
\[ y = \frac{1}{x} = x^{-1} \]

cc up

cc down

No point of inflexion

\[ f'(x) = \frac{1}{x^2} \]

\[ y' = -x^2 = -\frac{1}{x^2} < 0 \]

Dec fnct.

\[ y'' = +2x^3 = \frac{2}{x^3} \]

If \( x < 0 \), \( y'' < 0 \) cc down

If \( x > 0 \), \( y'' > 0 \) cc up
Week
3. 3
3. 4

Moe assigned