\[ y = 3 - |t-3| \quad \{1, 5\} \]

Critical #1

\[ y = \begin{cases} 3 - (t-3) & t \geq 3 \\ 3 - \left(-\frac{t-3}{2}\right) & t < 3 \end{cases} \]

\[ y = \begin{cases} 3 - t + 3 & t \geq 3 \\ 3 + t - 3 & t < 3 \end{cases} \]

\[ y = \begin{cases} 3 - t & t \geq 3 \\ t & t < 3 \end{cases} \]

\[ y' = \begin{cases} -1 & t > 3 \\ 1 & t < 3 \end{cases} \]

**Critical # \( t = 3 \)**

\[ f(3) = 3 - |3-3| = 3 - 0 = 3 \quad \text{max} \]

\[ f(-1) = 3 - |-1-3| = 3 - 4 = -1 \quad \text{min} \]

\[ f(5) = 3 - |5-3| = 3 - 2 = 1 \]

\[ \text{Max @ (3, 3)} \quad \text{Min @ (-1, -1)} \]
Rolle's theorem: If a function \( f \) is continuous on \([a, b]\) and differentiable on \((a, b)\) and \(f(a) = f(b)\) then there exists \(c \in (a, b)\) such that \(f'(c) = 0\).
\( f(x) = -3x \sqrt{x+1} \)

\( x = \text{int} \quad \text{let } y = 0 \)

0 = -3x \sqrt{x+1} \\
0 = x \sqrt{x+1} \\
x = 0 \\
\sqrt{x+1} = 0 \\
x+1 = 0 \\
x = -1 \\
\text{(c)}

3) \( h(x) = 0 \)

\( h(-1) = 0 \)

Consider \([-1, 0]\)

1) \( f \) is continuous on \([-1, 0]\) \( \forall x \in [-1, 0] \)

2) \( f \) is differentiable on \((-1, 0)\)

\[ f(x) = -3x \sqrt{x+1} \]
\[ f'(x) = -\frac{3x(x+1)}{(x+1)^{3/2}} \]
\[ f'(x) = \frac{-3\left(3x+2\right)}{2(x+1)^{3/2}} \]
\[ f'(-1) \text{ DNE} \]
\[ x = -1 \text{ is a critical point} \]

For continuity on \((-1, 0)\)

The three hypotheses of Rolle's theorem are satisfied. So there exists \( c \in (-1, 0) \) such that \( f'(c) = 0 \).

\[ f(c) = \frac{-3\left(3c+2\right)}{2(c+1)^{5/2}} \]
\[ 2(c+1)^{5/2} = 0 = \frac{-3\left(3c+2\right)}{2(c+1)^{5/2}} \]
\[ 0 = -3(3c+2) \]
\[ 0 = -9c - 6 \]
\[ 9c = -6 \]
\[ c = -\frac{2}{3} \]

Check:
Tangent line at $x = c$ is parallel to the secant line connecting $(a, f(a))$ to $(b, f(b))$

Subcase $f(a) + f(b)$

1) $f$ continuous on $[a, b]$
2) $f$ differentiable on $(a, b)$

Tangent at some point $x = c$ will be parallel to the secant line connecting $(a, f(a))$ to $(b, f(b))$

$$\frac{f(b) - f(a)}{b - a}$$

Mean Value Theorem

If $f$ is continuous on $[a, b]$ and differentiable on $(a, b)$, then there exists a point $c \in (a, b)$ such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$
\[ f(x) = x(x^2 - x - 2) \quad \text{on } [1, 2] \]
\[ f(x) = x^3 - x^2 - 2x \]

1. f continuous \([-1, 1]\) Polynomial yes
2. f differentiable \([-1, 1]\)

\[ f'(x) = 3x^2 - 2x - 2 \] Polynomial

So the hypotheses of the mean value theorem are satisfied, so there exists a \( c \in (-1, 1) \) such that

\[ f'(c) = \frac{f(b) - f(a)}{b - a} \]

\[ 3c^2 - 2c - 2 = \frac{f(1) - f(-1)}{1 - (-1)} \]

\[ 3c^2 - 2c - 2 = \frac{-2 - 0}{2} \]

\[ f(0) = 1(1-2) = -1 \]
\[ f(b) = f(1) = 2 \]
\[ f(-1) = -1(-1-2) = -3 \]
\[ f(1) = 0 \]
\[ f(-1) = 0 \]

\[ 3c^2 - 2c - 2 = -1 \]
\[ 3c^2 - 2c - 1 = 0 \]

\[ (c - 1)(3c + 1) = 0 \]
\[ c = 1, \quad 3c + 1 = 0 \]

\[ c = 1 \quad \text{or} \quad c = -\frac{1}{3} \]

\[ c = -\frac{1}{3} \]
Monday

3.2 Do those assigned

Read 3.3

3.4