\[ x^2 + y^2 = 2xy \]

Solve for y:

\[ y^2 - 2xy + x^2 = 0 \]
\[ ay^2 + by + c = 0 \]

\[ a = 1 \quad b = -2x \quad c = x^2 \]

\[ y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

\[ y = \frac{2x \pm \sqrt{4x^2 - 4(1)(x^2)}}{2(1)} \]
\[ y = \frac{2x \pm \sqrt{0}}{2} \]
\[ y = \frac{2x}{2} \]
\[ y = x \]

\[ \frac{dy}{dx} = 1 \]
Chapter 3

Extreme Values of a Func.

Maximums

Minimums

Def: $f(c)$ is a maximum value of $f$ on $I$ if $f(c) \geq f(x)$ for all $x$ in $I$.

Def: $f(c)$ is a minimum value of $f$ on $I$ if $f(c) \leq f(x)$ for all $x$ in $I$.

The Extreme Value Theorem

If $f$ is continuous on $[a, b]$, then $f$ has a maximum and a minimum value on $[a, b]$.

$f(c)$ is max, $f(b)$ is min.
Def. Let $f$ be defined at $x = c$.

If $f'(c) = 0$ or if $f'(c)$ DNE, the $c$ is called a **critical number** of $f$. 

- $f(c)$ exists.
- $f'(c) = 0$.
- No Extreme $b^+$. 

- $f(c)$ exists.
- $f'(c)$ DNE.
- No Extreme $b^-$. 

- $f(c) = f'(c)$ is Max.
- $f'(c)$ is Min.
- Vertical tangent.
- $f'(c)$ is Max.
- $f'(c)$ is Min.
- Vertical tangent.
- $f'(c) = 0$.
- No tangent.
In 3.2

Relative Extrema Occur Only at Critical #s.

If \( f \) has a relative maximum or minimum at \( x = c \), then \( c \) is a critical # of \( f \).
Critical #s $c = 0$, $f'(c) = 0$

Not an extreme point

Additional question

Domain $(-1,1)$

No absolute max or absolute min (on the domain)
Minimum in $(a, b)$? Yes! @ $x = c$
Maximum in $(a, b)$? No!

Ex: Fill in ends $f$

$f$ is continuous on $[a, b]$, so $f$ has a Max and a Min on $[a, b]$.

Extreme Value Theorem
Max @ $a$
Min @ $c$
Find the critical numbers.

\[ f(x) = \frac{4x}{x^2+1} \]  

\[ f'(x) = \frac{(x^2+1) \cdot 4 - 4x \cdot (2x)}{(x^2+1)^2} = \frac{-8x^2}{(x^2+1)^2} \]

\[ f''(x) = \frac{-8x^2 - 4 \cdot (x^2-1)}{(x^2+1)^2} = \frac{-4(x^2-1)}{(x^2+1)^2} \]

Set \( f'(x) = 0 \):

\[ (x^2+1)^2 \cdot 0 = \frac{-4(x^2-1)}{(x^2+1)^2} \cdot (x^2+1)^2 \]

\[ 0 = -4(x^2-1) \]

\[ 0 = x^2 - 1 \]

\[ x^2 = 1 \]

\[ x = \pm 1 \]

Critical numbers: \( x = \pm 1 \)

Horizontal tangent line:

Max at \( x = 1 \) \( f(1) = 2 \)

Min at \( x = -1 \) \( f(-1) = -2 \)

\( 2 \) is absolute max

\( -2 \) is absolute min.
P(6) Extreme Values of functions on a closed interval, \( [a, b] \):

1. Find all critical #s
2. Evaluate \( f(\text{critical #s}) \)
3. Evaluate \( f(a), f(b) \)
4. Largest in \( 2\) is Max.
   Smallest in \( 2\) is Min.
(2) Locate absolute extrema on \([a, b]\)

\(f(x) = x^3 - 12x\) on \([0, 4]\)

\text{Step 1: } \quad f'(x) = 3x^2 - 12

\begin{align*}
0 &= 3x^2 - 12 \\
3x^2 &= 12 \\
x^2 &= 4 \\
x &= \pm 2
\end{align*}

\(x = \pm 2\) are critical points.

\text{Step 2: } \quad f(2) = 2^3 - 12(2) = 8 - 24 = -16 \quad \text{Minimum @ } x = 2

\text{Step 3: } \quad f(0) = 0^3 - 12(0) = 0

\begin{align*}
f(4) &= 4^3 - 12(4) \\
f(4) &= 64 - 48 = 16
\end{align*}

\(x = 4\) is a maximum.
Friday 3.1 Those assigned

Read 3.2

Roll's th

Mean Value th