a) Guess's
\[
E = P(\text{correct}) (\text{gain}) + P(\text{wrong}) (\text{loss})
\]
\[
E = \frac{1}{3} (15) + \frac{2}{3} (-1)
\]
\[
E = \frac{5}{3} - \frac{2}{3}
\]
\[
E = \frac{1}{3}
\]
It is to your advantage to guess.

If you guessed on many, many questions, you could expect to gain on the average \( \frac{1}{3} \) bit per question.

b) Eliminates one choice
\[
E = P(\text{correct}) (\text{gain}) + P(\text{wrong}) (\text{loss})
\]
\[
E = \frac{1}{4} (15) + \frac{3}{4} (-1)
\]
\[
E = \frac{15}{4} - \frac{3}{4} = \frac{12}{4} = \frac{3}{2}
\]
If you do this many, many times, then you can expect to gain \( \frac{3}{2} \) bit on the average per question guessed upon.
\[ \frac{12}{35} \]$ to play

\[
E = P(1)(\text{net gain}) + P(5)(\text{net gain}) + P(10)(\text{net gain})
\]

\[
E = \frac{2}{4} (-1) + \frac{1}{4} (13) + \frac{1}{4} (18)
\]

\[
E = \frac{-2 + 3 + 8}{4} = \frac{9}{4} = 2.25
\]

If I play many times, I would expect to gain an average of $2.25 per play.

b) Fair Price = Cost + Expectation

\[
\text{Fair Price} = 2 + 2.25
\]

\[
\text{Fair Price} = \$4.25
\]
12.5 Tree Diagrams

Ex. Roll a die once, then flip a fair coin once. Find the probability that you get a number greater than 4 and a tail.

Sample Space: set of all possible outcomes.

Sample Space: \{\text{H, T}, 2\text{H, 2T}, 3\text{H, 3T}, 4\text{H, 4T}, 5\text{H, 5T}, 6\text{H, 6T}\}

Possibilities:

\[ P(\# > 4 \text{ and } \text{Tail}) = \frac{2}{12} = \frac{1}{6} \]
The Counting Principle

If a first task can be done in $M$ ways and a second task can be done in $N$ ways, then the sequence followed by the second can be done in $M \cdot N$ ways.
# 13 without replacement

C - circle
S - square
T - triangle

2 cards selected at random without replacement.

a) \(3 \cdot 2\) = 6 ways

b) 

\[ \text{Sample Space: } \{CS, CT, SC, ST, TC, TS\} \]

\[ P(CC) = \frac{1}{6} \]

\[ P(ST) = \frac{1}{6} \]

Ex. \( P(\text{square and triangle}) = \frac{2}{6} = \frac{1}{3} \)

\( P(\text{at least 1 circle}) = \frac{4}{6} = \frac{2}{3} \)
Two events are mutually exclusive if they can NOT happen at the same time.

Ex. Roll a fair coin once:
- Get a Head
- Get a Tail
These 2 are mutually exclusive events.

Ex. Draw one card from a standard deck:
- Get an Ace
- Get a King
These are mutually exclusive events.

Ex. Draw one card from a standard deck:
- Get an Ace
- Ace of Hearts
- Get a Heart
These 2 can happen at the same time. They are NOT mutually exclusive events.
Ex. Draw one card from a standard deck.

\[ P(\text{Ace or Heart}) = \frac{16}{52} = \frac{4}{13} \]

\[
\begin{array}{c}
\text{4 Aces} \\
\text{13 Hearts}
\end{array}
\]

\[
P(\text{A or B}) = P(\text{A}) + P(\text{B}) - P(\text{A and B})
\]

\[
P(\text{Ace or Heart}) = P(\text{Ace}) + P(\text{Heart}) - P(\text{Ace and Heart})
\]

\[
P(\text{Ace or Heart}) = \frac{4}{52} + \frac{13}{52} - \frac{1}{52}
\]

\[
P(\text{Ace or Heart}) = \frac{16}{52} = \frac{4}{13}
\]
Week

12.5 These assigned

12.6 3, 11

17-25 odd

Read 12.6