Mat 011 Agenda Day 21   June 21, 2006

Applications, PowerPoint 32
Worksheet
Quiz on Factoring

Homework: Topics 33 & 34
\[ y = -x^2 + 2x + 8 \]

**X-intercepts**

\[ 0 = -1x^2 + 2x + 8 \]

\[ 0 = -1(x^2 - 2x - 8) \]

\[ 0 = -(x + 2)(x - 4) \]

\[ x + 2 = 0 \quad \text{or} \quad x - 4 = 0 \]

\[ x = -2 \quad \text{or} \quad x = 4 \]

\[ (-2, 0) \quad (4, 0) \]

**Y-intercept**

\\[\text{Let } x = 0\]

\[ y = -1x^2 + 2x + 8 \]

\[ y = -1(0)^2 + 2(0) + 8 \]

\[ y = 8 \]
Enclose a rectangle with a 40 inch string and use a wall of the room for one side of the rectangle. Make a table and find the formula for the area of the rectangle.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
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<tbody>
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Wall

W

L

W
Enclose a rectangle with a 40 inch string and use a wall of the room for one side of the rectangle. Make a table and find the formula for the area of the rectangle.

<table>
<thead>
<tr>
<th>Width</th>
<th>Length</th>
<th>Area</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>40-2(5)=30</td>
<td>5(30)=150</td>
</tr>
<tr>
<td>10</td>
<td>40-2(10)=20</td>
<td>10(20)=200</td>
</tr>
<tr>
<td>15</td>
<td>40-2(15)=10</td>
<td>15(10)=150</td>
</tr>
<tr>
<td>X</td>
<td>40-2(X)</td>
<td>X(40-2X)=</td>
</tr>
<tr>
<td></td>
<td></td>
<td>40X-2X²</td>
</tr>
</tbody>
</table>

\[
A = x (40-2x)
\]
\[
A = 40x - 2x^2
\]
\[
A = -2x^2 + 40x
\]
Let \( x = \text{width} \)

\[
\begin{align*}
A &= x(40 - x) \\
\frac{40 - 3x}{2} &= 20 - x \\
A &= x \left( \frac{40 - 3x}{2} \right) \\
&= 40x - 3x^2 \\
A &= -\frac{3}{2}x^2 + 20x
\end{align*}
\]
\[ A = x(40 - 3x) \]
\[ A = 40x - 3x^2 \]
\[ A = -3x^2 + 40x \]
\[ A = -2x^2 + 40x \]

1. Opens down
2. Vertex: \( x = \frac{-b}{2a} = \frac{-40}{2(-2)} = \frac{-40}{-4} = 10 \)

(10, 200)

\[ A = -2x^2 + 40x \]

3. \( x \)-intercepts:
   
   \[ 0 = -2x^2 + 40x \]
   
   \[ 0 = -2x(x - 20) \]
   
   \(-2x = 0 \) or \( x - 20 = 0 \)

   \[ x = 0 \] or \( x = 20 \)

4. \( y \)-intercept: \( A = -2(0)^2 + 40(0) \)

   \[ = 0 \]
\[ A = -2x^2 + 40x \]

\[
\begin{array}{c|c}
 x & A \\
\hline
 10 & 260 \\
 20 & 0 \\
 0 & 0 \\
\end{array}
\]

Graph with points labeled:
- (10, 260)
- (18, 65)
- (1.8, 65)

Area calculation:
\[ \text{Area} = 20 \times 10 = 200 \text{ m}^2 \]
\[
\begin{align*}
ax^2 + bx + c &= 0 \
\Rightarrow x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\
A &= -2x^2 + 40x \\
V &= 40 \\
65 &= -2x^2 + 40x \\
C &= -65 \\
-65 &= -65 \\
0 &= -2x^2 + 40x - 65 \\
\chi &= \frac{-40 \pm \sqrt{1600 - 4(-2)(-65)}}{2(-2)} \\
&= \frac{-40 \pm \sqrt{1600 - 520}}{-4} \\
&= \frac{-40 \pm \sqrt{1080}}{-4} \\
&= \frac{-40 \pm 32.9}{-4} \\
&= 40 + 32.9, 40 - 32.9 \\
&= 18.2, 0.8
\end{align*}
\]
Enclose a rectangle with a 40 inch string and use a wall of the room for one side of the rectangle. Find the dimensions of the rectangle if the area is 65 square inches.

\[ A = 40x - 2x^2 \]

\[ 65 = -2x^2 + 40x \]
Enclose a rectangle with a 40 inch string and use a wall of the room for one side of the rectangle. Find the dimensions of the rectangle if the area is 65 square inches.

$$A = 40x - 2x^2$$

$$65 = -2x^2 + 40x$$

$$0 = -2x^2 + 40x - 65$$

Subtract 65 from both sides.
Use the Quadratic Formula to solve:

\[ 0 = -2x^2 + 40x - 65 \]
Use the Quadratic Formula to solve:

\[ 0 = -2x^2 + 40x - 65 \]

\[ x = \frac{-280 \pm 238}{-2} \]

\[ x = \frac{-40 \pm 32.86}{-4} \]

Two solutions to make A = 65 sq in:
\[ x = 1.8 \text{ and } x = 18.2 \text{ inches} \]
A 10 cm stick is broken into two pieces. One is placed at a right angle to form an upside down "T" shape. By attaching wires from the ends of the base to the end of the upright piece, a framework for a sail will be formed.

<table>
<thead>
<tr>
<th>Base</th>
<th>Height</th>
<th>Area (A=.5bh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>10 - 2 = 8</td>
<td>( \frac{1}{2} \times 2 \times 8 = 8 )</td>
</tr>
<tr>
<td>4</td>
<td>10 - 4 = 6</td>
<td>( \frac{1}{2} \times 4 \times 6 = 12 )</td>
</tr>
<tr>
<td>6</td>
<td>10 - 6 = 4</td>
<td>( \frac{1}{2} \times 6 \times 4 = 12 )</td>
</tr>
<tr>
<td>8</td>
<td>10 - 8 = 2</td>
<td>( \frac{1}{2} \times 8 \times 2 = 8 )</td>
</tr>
<tr>
<td>10</td>
<td>10 - 10 = 0</td>
<td>0</td>
</tr>
<tr>
<td>b</td>
<td>10 - b</td>
<td>( \frac{1}{2} \times b \times (10 - b) )</td>
</tr>
</tbody>
</table>

\[
A = \frac{1}{2} \times b \times (10 - b) = \frac{1}{2} \times 10b - \frac{b^2}{2} = 5b - \frac{b^2}{2}
\]
What should the base of the sail be if the area must be 10 sq cm?

\[ A = 5b - 0.5b^2 \]

10 = 5b - 0.5b^2

\[ 0 = -0.5b^2 + 5b - 10 \]

\[ b = \frac{5 \pm \sqrt{25 - 4(-0.5)(-10)}}{2(-0.5)} \]

\[ b = \frac{-5 \pm \sqrt{25 - 20}}{-1} \]

\[ b = \frac{-5 \pm \sqrt{5}}{-1} \]

\[ b = \frac{-5 \pm 2.2}{-1} \]

\[ a = -0.5 \]

\[ c = 5 \]

\[ e = -10 \]
Use the Quadratic Formula to solve:

\[ 0 = -0.5x^2 + 5x - 10 \]

\[ x = \frac{-5 \pm 2.24}{-1} \]

\[ x = \frac{-5 \pm 2.24}{-1} \]

Two solutions to make \( A = 10 \) sq in:
\( x = 2.8 \) and \( x = 7.2 \) inches
Graph $y = ax^2 + bx + c$.

Graph of a quadratic in the form $y = ax^2 + bx + c$ is a parabola.

Graph of $y = x^2 - 6x + 8$ is a parabola which opens upward because $a$ is positive coefficient.

Graph of $y = -x^2 + 2x + 8$ is a parabola which opens downward because $a$ is negative coefficient.
Graph $y = ax^2 + bx + c$.

Graph of a quadratic in the form $y = ax^2 + bx + c$ is a parabola.

Graph of $y = x^2 - 6x + 8$ is a parabola which opens upward because $a$ is positive coefficient. Think of a happy face!

Graph of $y = -x^2 + 2x + 8$ is a parabola which opens downward because $a$ is negative coefficient. Think of an unhappy face!
<table>
<thead>
<tr>
<th>Graph $y = ax^2 + bx + c.$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The vertex of the parabola $y = ax^2 + bx + c$ has x-coordinate $x = \frac{-b}{2a}.$</td>
</tr>
</tbody>
</table>
Graph \( y = ax^2 + bx + c \).

- The **vertex** of the parabola \( y = ax^2 + bx + c \) has x-coordinate \( x = \frac{-b}{2a} \).

- The **x-coordinate** for vertex of \( y = -1x^2 + 2x + 8 \) is \( x = \frac{-(2)}{2(-1)} = \frac{2}{2} = 1 \).

- To find **y-coordinate** for vertex of \( y = -1x^2 + 2x + 8 \), substitute the x value in the equation and evaluate y.
  \[ y = -1(1)^2 + 2(1) + 8 = -(1)^2 + 2(1) + 8 = 9 \]

- The **vertex** (highest point) on this parabola is \((1, 9)\).
Graph $y = ax^2 + bx + c$.

- The **vertex** of the parabola $y = ax^2 + bx + c$ has x-coordinate $x = \frac{-b}{2a}$.
- The **x-coordinate** for vertex of $y = -1x^2 + 2x + 8$ is $x = \frac{-2}{2(-1)} = \frac{-2}{-2} = 1$.
- To find y-coordinate for vertex of $y = -1x^2 + 2x + 8$, substitute the x value in the equation and evaluate y.
  - $y = -1(1)^2 + 2(1) + 8 = -1 + 2 + 8 = 9$
- The vertex (highest point) on this parabola is $(1, 9)$. 
Graph \( y = ax^2 + bx + c \).

To find \textit{x-intercepts}, let \( y=0 \) and solve for \( x \).
To find \textit{y-intercept}, let \( x=0 \) and solve for \( y \).

Find the \textit{x-intercepts} for \( y = x^2 - 6x + 8 \)

Let \( y = 0 = x^2 - 6x + 8 \)
\( 0 = (x-4)(x-2) \)
\( x-4 = 0 \) or \( x-2 = 0 \)
\( x = 4 \) or \( x = 2 \)

\textit{x-intercepts:} \((4,0)\) \((2,0)\)

Find the \textit{y-intercept} for \( y = x^2 - 6x + 8 \)

Let \( x = 0 \)
\( y = (0)^2 - 6(0) + 8 = 8 \)

\textit{y-intercept:} \((0,8)\)
Graph \( y = ax^2 + bx + c \).

To find \textbf{x-intercepts}, let \( y = 0 \) and solve for \( x \).

To find \textbf{y-intercept}, let \( x = 0 \) and solve for \( y \).

\[ \text{Find the x-intercepts for } y = -1x^2 + 2x + 8 \]

Let \( y = 0 = -1x^2 + 2x + 8 \)

\[ 0 = -(x-4)(x+2) \]

\[ (x-4) = 0 \text{ or } (x+2) = 0 \]

\[ x = 4 \text{ or } x = -2 \]

x-intercepts: (4,0) (-2,0)

\[ \text{Find the y-intercept for } y = -1x^2 + 2x + 8 \]

Let \( x = 0 \)

\[ y = -(0)^2 + 2(0) + 8 = 8 \]

y-intercept: (0,8)
Graph \( y = x^2 - 6x + 8 \).
Graph \( y = x^2 - 6x + 8 \).

Summary: \( y = x^2 - 6x + 8 \).

Graph of \( y = 1x^2 - 6x + 8 \) is a parabola which opens **upward** because \( a \) is positive coefficient.

The vertex (lowest point) on this parabola is \((3, -1)\).

x-intercepts: \((4,0)\) \((2,0)\)

y-intercept: \((0,8)\)
Graph \( y = -x^2 + 2x + 8 \).
Graph \( y = -x^2 + 2x + 8 \).

**Summary:** \( y = -x^2 + 2x + 8 \).

Graph of \( y = -1x^2 + 2x + 8 \) is a parabola which opens **downward** because \( a \) is negative coefficient.

The vertex (highest point) on this parabola is \((1, 9)\).

x-intercepts: \((4,0)\) \((-2,0)\)

y-intercept: \((0,8)\)
\[ x^2 + 7x + 12 = (x+4)(x+3) \]

\[ x^2 + 13x + 12 = (x+1)(x+12) \]
3. \(x^2 - 8x + 12 = (x - 6)(x - 2)\)

4. \(x^2 - 7x + 12 = (x - 4)(x - 3)\)
5. \( x^2 + 4x - 12 = (x+6)(x-2) \)

6. \( x^2 - 4x - 12 = (x-6)(x+2) \)
\( 7 \) \( x^2 + 1x - 12 \)
\[ (x-3)(x+4) \]

\( 8 \) \( x^2 - x - 12 \)
\[ (x+3)(x-4) \]
\( \alpha^2 + 8\alpha + 16 = (\alpha + 4)^2 \)

\( \alpha^2 + 8\alpha - 12 = 0 \)
\[ a = 1 \quad b = 8 \quad c = -12 \]
\[ \alpha = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]
\[ \alpha = \frac{-8 \pm \sqrt{64 + 48}}{2} \]
\[ \alpha = \frac{-8 \pm \sqrt{112}}{2} \]
\[ \alpha = \frac{-8 \pm 10.6}{2} \]
\[ \alpha = \frac{-8 - 10.6}{2} = \frac{-18.6}{2} = -9.3 \]
\[ \alpha = \frac{-8 + 10.6}{2} = \frac{2.6}{2} = 1.3 \]
\[ \alpha = 1.3 \quad \alpha = -9.3 \]
\[ (\alpha - 1.3)(\alpha + 9.3) = 0 \]
(11) \( \alpha^2 - 16 = (\alpha - 4)(\alpha + 4) \)

(12) \( \alpha^2 - 16 \alpha = \alpha (\alpha - 16) \)
Earl Black makes tea bags. The cost of making x million teas bags per month is $C = x^2 - 38x + 400$. The revenue from selling x million tea bags per month is $R = 78x - x^2$

Find the equation for Profit

$$\text{Profit} = \text{Revenue} - \text{Cost}$$

Revenue: $R = 78x - x^2$

Cost: $C = x^2 - 38x + 400$
The equation for profit is,

\[ \text{Profit} = \text{Revenue} - \text{Cost} \]

Revenue: \[ R = 78x - x^2 \]

Cost: \[ C = x^2 - 38x + 400 \]

\[ P = (78x - x^2) - (x^2 - 38x + 400) \]
Graph the Profit Equation

\[ P = -2x^2 + 116x - 400 \]

**Vertex:**

\[ x = \frac{-b}{2a} \]

\[ x = \frac{-116}{2(-2)} = \frac{-116}{-4} = 29 \]

\[ P = -2(29)^2 + 116(29) - 400 = 1282 \]

**Vertex:** \((29, 1282)\)

\[ x = 29, \ P = \$1282 \]
Worksheet

If the cost function for making coffee makers is
\[ C = .2x^2 - 15x + 75 \] and
the revenue for selling them is:
\[ R = -.8x^2 + 5x \]
\( C \) and \( R \) are in hundreds of dollars.
Find the profit equation, and graph.
Ms. Piggie wants to enclose two adjacent chicken coops of equal size against the hen house wall. She has 66 feet of chicken-wire fencing and would like to make the chicken coup as large as possible. Find the formula for the area of the chicken coops.
Graph: \[ A = -3X^2 + 66X \]

1. Because \( a = -3 \), parabola opens down
2. Vertex: \((11, 363)\)
3. \( x \)-Intercepts: \((0, 0)\) and \((22, 0)\)
4. \( A \)-Intercept: \((0, 0)\)
Graph:

\[ A = -3X^2 + 66X \]

Area (sq. feet)

(0,0) (11,363) (22,0)

Width (feet)
Find the dimensions of the coop if the area can only be 360 sq feet.
Graph: \[ A = -3X^2 + 66X \]

- Points: (0,0), (11,363), (12,360), (22,0)
- Area (sq. feet): 350, 300, 250, 200, 150, 100, 50
- Width (feet): 3, 6, 9, 12, 15, 18, 21
Graph: \[ A = - 3X^2 + 66X \]

- Point (0,0)
- Point (3,360)
- Point (10,360)
- Point (11,363)
- Point (12,360)
- Point (15,360)
- Point (22,0)

Area (sq. feet)

Width (feet)
Factor

\[ x^2 - 9x + 8 \]

\[ x^2 - 2x - 8 \]

\[ x^2 - 6x + 8 \]

\[ x^2 - 36x \]

\[ x^2 - 36 \]
Lighten Up Company makes light bulbs. The cost of making $x$ thousand light bulbs per week is $C = 0.5x^2 - 14x + 120$. The revenue from selling $x$ thousand light bulbs per week is $R = 12x - 0.5x^2$

Find the equation for Profit
A farmer wants to enclose adjacent rectangular fields with 1000 feet of barbed wire fencing as indicated below. Find the equation for the area of the fields.