Mat 011 Agenda Day 12   June 6, 2006

Return Test 2
Positive Exponents, PowerPoint 21
Negative Exponents, PowerPoint 22
Properties of Exponents, PowerPoint 23

Homework:  Topics 19, 20, 21
Objectives

- To evaluate exponential expressions with positive exponents
- To learn how to apply an interest formula to problem situation
- To learn how to apply an exponential growth formula to a population situation
Raising a Number to a Power

\[ b^n \] means multiply \( b \) by itself \( n \) times:

\[ b \times b \times b \times \ldots \times b \ (n \text{ times}) \]

\( n \) is the power

\( b \) is the base

\[ 3^4 \] means multiply 3 by itself 4 times:

\[ 3^4 = 3 \times 3 \times 3 \times 3 = 81 \]
$2^4 = 2 \cdot 2 \cdot 2 \cdot 2$

$b^n = \underbrace{b \cdot b \cdot b \cdots b}_{n}$
Use of the calculator to evaluate an exponential expression.
To raise a number to a power use the \(^\text{key}\),
5\(^{\text{th}}\) row, 1\(^{\text{st}}\) column.
Use of the calculator to evaluate an exponential expression.

To raise a number to a power use the

^ key,

5th row, 1st column.

$3^4$ is keyed in as

3, ^, 4, ENTER
Use of the calculator to evaluate an exponential expression.

To square a number use the $x^2$ key, 6th row, 1st column.
Use of the calculator to evaluate an exponential expression.

To square a number use the \( x^2 \) key, 6th row, 1st column.

\( 3^2 \) is keyed in as 3, \( x^2 \), ENTER
Use of the calculator to evaluate an exponential expression.
To evaluate a negative expression use the (-) key,
9th row, 4th column.

\[-3^2 = \left(-3\right)\left(-3\right) = 9\]
Use of the calculator to evaluate an exponential expression.
To evaluate a negative expression use the (-) key,
9\textsuperscript{th} row, 4\textsuperscript{th} column.
\(-3^2\) is keyed in as
(-), 3, \textit{x}^2, \text{ENTER}
Use of the calculator to evaluate an exponential expression.
To evaluate \((-3)^2\) use the ( key, and ) key

5\textsuperscript{th} row, 3\textsuperscript{rd} and 4\textsuperscript{th} column.
Use of the calculator to evaluate an exponential expression.

To evaluate \((-3)^2\) use the \((\text{key}, \text{ and } \text{key})\) key

5\(^{th}\) row, 3\(^{rd}\) and 4\(^{th}\) column.

\((-3)^2\) is keyed in as \((, (-), 3, ), x^2, \text{ENTER}\)
<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^4 = $</td>
<td>81</td>
</tr>
<tr>
<td>$3^2 = $</td>
<td>9</td>
</tr>
<tr>
<td>$-3^2 = $</td>
<td>-9</td>
</tr>
<tr>
<td>$(-3)^2 = $</td>
<td>9</td>
</tr>
<tr>
<td>Expression</td>
<td>Evaluation</td>
</tr>
<tr>
<td>-----------------</td>
<td>------------</td>
</tr>
<tr>
<td>((-6)^2)</td>
<td>(36)</td>
</tr>
<tr>
<td>((-8.2)^4)</td>
<td>(4521.28)</td>
</tr>
<tr>
<td>(7^0)</td>
<td>(1)</td>
</tr>
<tr>
<td>(-6^2)</td>
<td>(-36)</td>
</tr>
<tr>
<td>Expression</td>
<td>Evaluation</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>(-8.2^4) =</td>
<td>(-4521.22)</td>
</tr>
<tr>
<td>(8.6^0) =</td>
<td>1</td>
</tr>
<tr>
<td>((-3.1)^3) =</td>
<td>(-29.791)</td>
</tr>
<tr>
<td>(-3.1^3) =</td>
<td>(-29.791)</td>
</tr>
</tbody>
</table>

\(\begin{align*}
(-3)^2 &= 9 \\
(-3)^3 &= -27
\end{align*}\)
A couple invests $2000 at an annual interest rate of 12%. How much money will they have after 10 years?

\[ FV = P(1 + i)^n \]

- **FV** = Future Value
- **P** = Amount invested
- **i** = Interest rate per month
- **n** = number of times compounded

\[
FV = 2000(1 + 0.12)^{10} \\
= 2000(1.12)^{10} \\
= 2000(3.1058) \\
= \$6211.69
\]
<table>
<thead>
<tr>
<th>Year</th>
<th>Amt</th>
<th>Interest</th>
<th>New Amt</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2,000</td>
<td>$240</td>
<td>$2,240</td>
</tr>
<tr>
<td>2</td>
<td>$2,240</td>
<td>$268.80</td>
<td>$2,508.80</td>
</tr>
<tr>
<td>3</td>
<td>$2,508.80</td>
<td>$301.06</td>
<td>$2,809.86</td>
</tr>
</tbody>
</table>

\[
2000 \times (1 + 0.12) = 2240.
\]

\[
2000 \times (1.12)^2 = 2508.80.
\]

\[
2000 \times (1.12)^3 = 2809.86.
\]
How much money should you invest at an annual interest rate of 6%, if you want $10,000 in 20 years?

\[ FV = P(1 + i)^n \]

- **FV** = Future Value
- **P** = Amount invested
- **i** = Interest rate per month
- **n** = number of times compounded
The population of Shanghai was 10,820,000 in 1974. If the population increased by 1% each year, complete the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td>10,820,000</td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td>10,820,000 + 108,200</td>
<td>10,828,200</td>
</tr>
<tr>
<td>1976</td>
<td>10,820,000 (1.01) = 10,928,200</td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td>11,147,856 (1.01)^2 = 11,437,949</td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td>11,289,335 (1.01)^3 = 11,803,749</td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>11,371,929 (1.01)^4 = 12,057,279</td>
<td></td>
</tr>
</tbody>
</table>

\[10,820,000 (1.01^n)\]
<table>
<thead>
<tr>
<th>Day</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1000 + 50$</td>
</tr>
<tr>
<td>2</td>
<td>$1050 + 52.50$</td>
</tr>
<tr>
<td>3</td>
<td>$1102.50 + 55.13$</td>
</tr>
<tr>
<td>30</td>
<td>$\frac{1550}{(1.05)^{30}} \approx 4322$</td>
</tr>
</tbody>
</table>

$1000 (1.05) = 1050$

$1000 (1.05)^2 = 1102.50$

$1000 (1.05)^3 \approx 1103$

$1000 (1.05)^{30} = 4321.94$

$\approx 4322$
Raising a Number to a Negative Power

$b^{-n}$ means $\frac{1}{b^n}$

$$b^{-n} = \frac{1}{b^n}$$

$$3^{-2} = \frac{1}{3^2} = \frac{1}{3 \cdot 3} = \frac{1}{9} = .\overline{1}$$

$$3^{-\frac{2}{1}} = .\overline{1}$$
Raising a Number to a Negative Power

\[ b^{-n} \text{ means } \frac{1}{b^n} \]

\[ b^{-n} = \frac{1}{b^n} = \frac{1}{b \cdot b \cdot b \cdots \cdot b} \]

\[ 3^{-4} \text{ means } \frac{1}{3^4} \]

\[ 3^{-4} = \frac{1}{3^4} = \frac{1}{3 \cdot 3 \cdot 3 \cdot 3} = \frac{1}{81} = .012345 \]
Use of the calculator to evaluate an exponential expression.
To raise a number to a negative power use the $^\wedge$ key, and (-) key.

$3^{-4}$ is keyed in as 3, $^\wedge$, (-), 4, ENTER.
<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8^{-2}$</td>
<td>0.0156</td>
</tr>
<tr>
<td>$-6^{-4}$</td>
<td>-0.00011605</td>
</tr>
<tr>
<td>$(-6)^{-4}$</td>
<td>7.716e-9</td>
</tr>
<tr>
<td>$0.08^{-5}$</td>
<td>305175.7812</td>
</tr>
</tbody>
</table>
Find the monthly payments on a 48-month car loan of $18,000 at 3% annual interest.

\[
P = \frac{A}{\frac{i}{1-(1+i)^{-n}}}
\]

\[
i = \frac{0.03}{12} = 0.0025
\]

\[
= 18,000 \left( \frac{0.0025}{1-(1.0025)^{-48}} \right)
\]

\[
= 18,000 \left( \frac{0.0025}{1-0.8871} \right)
\]

\[
= 18,000 \left( \frac{0.0025}{0.1129} \right)
\]

\[
= 18,000 \cdot 0.2213
\]

\[
= 398.52
\]
Clinton's can afford a $1000 monthly house payment at 7.2% annual interest rate for 360 months. How expensive a house can they afford?

\[
P = A \left( \frac{i}{1-(1+i)^{-n}} \right)
\]

\[
0.72\% = \frac{0.072}{12} = 0.006
\]

\[
1000 = A \left( \frac{0.006}{1 - (1.006)^{-360}} \right)
\]

\[
1000 = A \left( \frac{0.006}{1 - 0.1161} \right)
\]

\[
1000 = A \left( \frac{0.006}{0.8839} \right)
\]

\[
1000 = A \left( 0.006788 \right)
\]

\[
\frac{1000}{0.006788} = A
\]

\[
149,320 = A
\]
Scientific Notation
A number in Scientific Notation has the form P \times 10^n where 1 \leq P < 10 and n is an integer.

8,200,000 = \begin{align*} &8.2 \times 10^6 \quad \text{or} \\ &8.2 \times 10^5 \times 10,000 \end{align*}
Scientific Notation

A number in Scientific Notation has the form $P \times 10^n$ where $1 \leq P < 10$ and $n$ is an integer.

\[8,200,000 = 8.20 \times 10^6\]
A number in Scientific Notation is written as $P \times 10^n$ where $1 \leq P < 10$.

$.000517 = \frac{5.17}{10^4} = \frac{5.17}{10000}$

$$5.17 \times 10^{-4} = \frac{5.17}{10^4} = \frac{5.17}{10000}$$
Scientific Notation

A number in Scientific Notation has the form $P \times 10^n$ where $1 < P < 10$ and $n$ is an integer.

$0.000517 =$
Scientific Form to Decimal Form

A number in Scientific Notation has the form $P \times 10^n$ where $1 \leq P < 10$ and $n$ is an integer.

$7.3 \times 10^6$
Scientific Form to Decimal Form

A number in Scientific Notation has the form \( P \times 10^n \) where \( 1 \leq P < 10 \) and \( n \) is an integer.

\[ 3.141 \times 10^{-4} \]
Scientific Notation

A number in Scientific Notation has the form $P \times 10^n$ where $1 \leq P < 10$ and $n$ is an integer.

$0.000517 =$
<table>
<thead>
<tr>
<th>Scientific</th>
<th>Decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>$8.14 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$4.18 \times 10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>$7.86 \times 10^8$</td>
<td></td>
</tr>
<tr>
<td>$8.673 \times 10^{-10}$</td>
<td></td>
</tr>
<tr>
<td>$3.3 \times 10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>Decimal</td>
<td>Scientific</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
</tr>
<tr>
<td>.0028</td>
<td></td>
</tr>
<tr>
<td>78,000</td>
<td></td>
</tr>
<tr>
<td>.00000167</td>
<td></td>
</tr>
<tr>
<td>.000635</td>
<td></td>
</tr>
<tr>
<td>1,160,000</td>
<td></td>
</tr>
<tr>
<td>Given</td>
<td>Changed Format</td>
</tr>
<tr>
<td>-------------</td>
<td>----------------</td>
</tr>
<tr>
<td>.0028</td>
<td></td>
</tr>
<tr>
<td>82000</td>
<td></td>
</tr>
<tr>
<td>$8.14 \times 10^3$</td>
<td></td>
</tr>
<tr>
<td>$4.18 \times 10^{-4}$</td>
<td></td>
</tr>
</tbody>
</table>
Use calculator to evaluate:

$$(6.3 \times 10^8)(4.2 \times 10^9)$$

To multiply a number in scientific notation use the EE key, 5th row, 2nd column.
Use calculator to evaluate:
\[(6.3 \times 10^8)(4.2 \times 10^9)\]

To multiply a number in scientific notation use the EE key, 5th row, 2nd column. 
\[(6.3 \times 10^8)(4.2 \times 10^9)\] is keyed in as 6.3, EE, 8, x, 4.2, EE, 9, Enter.
Multiplication

\[ a^n \times a^m = a^{n+m} \]

When you multiply, add the exponents as long as the bases are the same.
\(2^5 \times 2^3\)

\(x^5 \times x^3\)
Multiplication

\[ a^n \times a^m = a^{n+m} \]

When you multiply, add the exponents as long as the bases are the same.

\[ 2^5 \times 2^3 = 2^{5+3} = 2^8 \]

\[ (2 \times 2 \times 2 \times 2 \times 2) \times (2 \times 2 \times 2) = 2^8 \]

\[ x^5 \times x^3 = x^{5+3} = x^8 \]
Raise an Expression to a Power

\((a^n)^m = a^{n \times m}\)

When you raise an expression to a power, multiply the exponents.
\[(2^5)^3 = \]
\[(2^5)^3 = \]
\[(x^3)^2 = \]
\[(3x^{2})^4 = \]
Division

\[ a^n / a^m = a^{n-m} \]

When you divide, subtract the exponents as long as the bases are the same.

\[ \frac{2^5}{2^3} = 2^{5-3} = 2^2 \]
\[ \frac{2^5}{2^3} \cdot \frac{2^5}{2^3} \cdot x^8 \cdot x^3 \]
Expression Raised to a Negative Power

\[ a^{-n} = \frac{1}{a^n} \]

When you raise an expression to a negative power, take the reciprocal of the expression.
\[ 8x^{-3} = (2x)^{-4} = \]
\[ \frac{1}{(4x^{-3})^3} \]
\[ \frac{1}{(7x^2)^{-3}} \]
\[(2x)^{-4}\]
\[8x^{-3}\]
\[1/(4x^{-3})\]
$\frac{1}{(7x^2)^{-3}}$
<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3x^2x^3$</td>
<td></td>
</tr>
<tr>
<td>$4(-3x^2)^2$</td>
<td></td>
</tr>
<tr>
<td>$6x(-2x^2)^3$</td>
<td></td>
</tr>
<tr>
<td>Expression</td>
<td>Evaluation</td>
</tr>
<tr>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>$\frac{8x^5}{x^{-3}}$</td>
<td></td>
</tr>
<tr>
<td>$4x^{-3}$</td>
<td></td>
</tr>
<tr>
<td>$\frac{8x^{-1}}{8x^{-1}}$</td>
<td></td>
</tr>
<tr>
<td>$\left(-\frac{2}{x^2}\right)^3$</td>
<td></td>
</tr>
</tbody>
</table>
The population of Shanghai was 10,820,000 in 1974. If the population increased by 1% each year, complete the table.

<table>
<thead>
<tr>
<th>Year</th>
<th>Calculation</th>
<th>Population</th>
</tr>
</thead>
<tbody>
<tr>
<td>1974</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1975</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1976</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1977</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Compute the following using a calculator:

1. $8^{-3}$

2. $-8^{-3}$

3. $(-8)^{-3}$

4. $(3.8 \times 10^5)(6.2 \times 10^7)$
5. $8x^5 x^3$

6. $(8x^3)^2$

7. $\left(\frac{x^4}{2}\right)^{-3}$
9. Given \( FV = P(1 + i)^n \), find the future value of $1500 deposit if the annual rate is 8% compounded monthly for 20 years. (Hint: \( i = \frac{0.08}{12} \) and \( n = 12 \times 20 \))
8. Use the formula  

\[ P = A \frac{i}{1 - (1 + i)^{-n}} \]

- \( P \) is the payment
- \( A \) is the amount of the loan
- \( n \) is the number of payments
- \( i \) is the interest rate per month

TJ Ridge is borrowing $15,000 to buy a car. He takes out a 36-month loan at 6% annual interest. (Hint: \( i = .06/12 \)). Find Tom's monthly payments.