Mat 011 Agenda Day 15       June 13, 2005
Ratio and Proportion, PowerPoint 26
Worksheets
Review for Test 3
Quiz
Homework:  Topic 24, 25
Unit 3 Lecture 26
Ratio and Proportion

Ratio and Proportion
<table>
<thead>
<tr>
<th><strong>Ratio:</strong></th>
<th>A quotient of 2 numbers or quantities.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratio of 2 parts soap to 3 parts water</strong></td>
<td>2:3 or $\frac{2}{3}$</td>
</tr>
<tr>
<td><strong>Proportion</strong></td>
<td>A mathematical statement that 2 ratios are equal.</td>
</tr>
<tr>
<td><strong>How parts of soap to 9 parts of water?</strong></td>
<td>$\frac{x}{9} = \frac{2}{3}$</td>
</tr>
<tr>
<td>( \frac{2}{9} )</td>
<td>( \frac{2}{3} )</td>
</tr>
<tr>
<td>-----------------</td>
<td>-----------------</td>
</tr>
<tr>
<td>( \frac{x}{9} )</td>
<td>Solve the equation for ( x )</td>
</tr>
</tbody>
</table>
### Ratio and Proportion

\[
\frac{6}{x} = \frac{5}{7}
\]

Solve the equation for \( x \)
To estimate the number of fish in a lake, 30 fish are caught, tagged and released. Later 70 fish are caught and 14 are found to have been tagged. Estimate the number of fish in the lake.

\[
\frac{14 \text{ tagged}}{30 \text{ tagged}} = \frac{70 \text{ sample}}{x \text{ population}}
\]

\[
\frac{\text{tagged}}{\text{pop}} = \frac{14}{70}
\]

\[
14x = 2100
\]

\[
x = 150 \text{ fish}
\]
Two people put their money together to buy lottery tickets. The first person puts in $15 and the second puts in $25. If they won 2.4 million dollars, how much does each person win?

\[
\frac{15}{40} = \frac{x}{2.4\text{ million}}
\]

\[
40x = 15(2.4)
\]

\[
40x = 36
\]

\[
x = 0.9\text{ million}
\]

\[
\frac{15}{25} = \frac{2.4\text{ million}}{x}
\]

\[
2.4\text{ million}
\]

\[
1.5\text{ million}
\]
\[ \frac{25}{40} = \frac{x}{2.4 \text{ million}} \]

\[ 40x = 60 \]

\[ x = 1.5 \text{ million} \]
Two people put their money together to buy lottery tickets. The first person puts in $15 and the second puts in $25. If they won 2.4 million dollars, how much does each person win?

The total amount put together is $15+$25 = $40. Let $x =$ amount each person wins.

\[
\frac{x}{2.4} = \frac{15}{40}
\]

Solve for $x$ for 1st person.
<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{8x^5}{x^{-3}}$</td>
<td>$8x^8$</td>
</tr>
<tr>
<td>$\frac{4x^{-3}}{8x^{-1}}$</td>
<td>$\frac{1}{2x^2}$</td>
</tr>
<tr>
<td>$\left(\frac{-2}{x^2}\right)^3$</td>
<td>$(-2)^3 \frac{-8}{x^6} = \frac{(-2)(-2)(-2)}{x^6}$</td>
</tr>
<tr>
<td>$\frac{(4x)^{-3}}{(8x)^{-1}}$</td>
<td>$\frac{8x^{3/3}}{4x^{1/3}} = \frac{8}{64x^{2/3}} = \frac{1}{8x}$</td>
</tr>
</tbody>
</table>
\[
\begin{align*}
\frac{2}{3} & \cdot 6x^2 (2x y')^3 \\
& \cdot (6x y')^2 \cdot (-8x^6 y^3) \\
& -48x^7 y^5
\end{align*}
\]

\[
(\chi^2)^3 = \chi^6 \quad \chi^2 \cdot \chi^3 = \chi^5
\]
\[ \frac{1}{4} \left( \frac{1}{(-3 + x^2)} \right)^2 = \frac{1}{4} \left( -3 \right)^2 \left( x^2 \right)^2 \]

\[ 4 \left( \frac{1}{(-3 + x^2)} \right)^2 = 4 \left( -3 \right)^2 \left( x^2 \right)^2 \]

\[ 4 \left( -3 \right)^2 x^4 \]

\[ 4 \left( 9 \right) x^4 \]

\[ 36 x^4 \]

\[ \left( 5 x^2 \right)^3 \]

\[ 125 x^6 \]

\[ \left( -3 x^2 \right)^2 = 9 x^4 \]
Distribute multiplication over addition

\[
4 \left( x^2 y^5 \right)^3 = 4 \times x^6 y^{15}
\]

\[
2 (3 + 4) = 24 \quad \text{and} \quad 6 \times 8 = 14
\]

\[
(4x^2 y^5)^3 = 64x^6 y^{15}
\]
\[ 2(3+4) \rightarrow 2(x+y) \]

\[ 2 \cdot 3 + 2 \cdot 4 \]

\[ 2x + 2y \]

Addition:
- \( a + b = b + a \)
- \( a + 0 = a \)

Multiplication:
- \( a \cdot b = b \cdot a \)
- \( a \cdot 1 = a \) (Identity)
Compute the following using a calculator:

1. $8^{-3}$
   
   $0.001953$

2. $-8^{-3}$
   
   $-0.001953$

3. $(-8)^{-3}$
   
   $-0.001953$

4. $(3.8 \times 10^5)(6.2 \times 10^7)$
   
   $2.356 \times 10^{12}$

   $\times$

   $2.356 \times 10^{13}$
5. \( 8x^5x^3 = 8x^8 \)

6. \((8x^3)^2 = 8^2x^6 = 64x^6 \)

7. \( \left( \frac{x^4}{2} \right)^{-3} = \left( \frac{2^1}{x^4} \right)^3 = \frac{2}{x^{12}} = \frac{8}{x^{12}} \)

\( \frac{x^{-12}}{2^{-3}} = \frac{2^3}{x^{12}} = \frac{8}{x^{12}} \)
9. Given $FV = P(1 + i)^n$, find the future value of $1500$ deposit if the annual rate is 8% compounded monthly for 20 years. (Hint: $i = \frac{0.08}{12}$ and $n = 12 \cdot 20$)

\[
FV = 1500 \left(1 + \frac{0.08}{12}\right)^{240}
\]

\[
= 1500 \left(1.0066667\right)^{240}
\]

\[
= 1500 \cdot 4.9272
\]

\[
= 7,390.80
\]
8. Use the formula \( P = \frac{A}{\left(1 + i\right)^n} \)

- \( P \) is the payment
- \( A \) is the amount of the loan
- \( n \) is the number of payments
- \( i \) is the interest rate per month

T.J. Ridge is borrowing $15,000 to buy a car. He takes out a 36-month loan at 6% annual interest. (Hint: \( i = \frac{0.06}{12} \)). Find Tom’s monthly payments.

\[
P = 15,000 \left( \frac{0.005}{1 - (1.005)^{-36}} \right) \\
= 15,000 \left( \frac{0.005}{1 - 0.8356} \right) \\
= 15,000 \left( \frac{0.005}{0.1644} \right) \\
= 15,000 \times 0.3041 \\
= \$456.15
\]
5. \(7x^4 x^3 = 7x^7\)

6. \((7x^4)^3 = 7^3 x^{12} = 343x^{12}\)
$8^{-4} = 0.0002441$

$-8^{-4} = -0.0002441$

$(-8)^{-4} = 0.0002441$

$(8.3 \times 10^8)(2.6 \times 10^6) = 2.158 \times 10^{15}$
7. \left( \frac{x^5}{2} \right)^{-3} = \frac{8}{x^{15}}
\[
\frac{x^{-3}}{(3x^4)^{-5}}
\]

\[
\frac{(3^5x^{20})}{x^3} = 243x^{17}
\]
9. \[ \frac{\frac{5x^{-3}}{20x^4}}{4} = \frac{1}{4x^4x^3} = \frac{1}{4x^7} \]

\[ \frac{(5x)^{-3}}{20x^4} = \frac{1}{(5x)^3 20x^4} = \frac{1}{125x^3 20x^4} = \frac{1}{2500x^7} \]
\[
\frac{x^3}{x^6} = x^{3-6} = x^{-3} = \frac{1}{x^3}
\]

\[
\frac{x \cdot x \cdot x}{x \cdot x \cdot x} = \frac{1}{x^3}
\]
Simplify: \( \frac{2}{9x} \div 4x^5 \)

Simplify: \( \frac{3}{4x^2} + \frac{8}{3x} \)
Simplify: $6x + \frac{4x}{9}$
Solve for $x$: \[ \frac{4x - 5}{3} - \frac{x - 2}{4} = -1\]
5. Solve for \( x \):
\[
\frac{x-4}{2} - \frac{x-3}{9} = \frac{5}{18}
\]
1. \( \frac{12x^2}{3x^6} \)
3. \((2x^3)^4\)
4. \[
\frac{x^{-5}}{(2x^2)^{-4}}
\]
2. \[ \frac{2x^{-2}}{10x^3} \]
5. Compute the following using a calculator and write in scientific notation:

\[
\left(8.2 \times 10^{11}\right)\left(3.6 \times 10^{-2}\right)
\]
Which of the following operations require that you find a LCD when you are simplifying algebraic fractions?

<table>
<thead>
<tr>
<th>Operation</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
</tr>
<tr>
<td>subtraction</td>
</tr>
<tr>
<td>multiplication</td>
</tr>
<tr>
<td>division</td>
</tr>
</tbody>
</table>
1. Find the LCD of the two fractions:

\[ \frac{1}{5x} + \frac{1}{35x^6} = \]
3. The LCD is $24x^2$. What do you have to multiply the numerator and denominator by to convert the following fraction?
\[
\frac{1}{3x}
\]
4. Describe what is wrong with the following problem.

Reduce: \( \frac{A + 5}{A} = 1 + 5 \)
4a. Describe what is wrong with the following problem.

Reduce: \( \frac{A + 5}{A} = 1 + 5 \)

You cannot cancel terms.

4b. What are you allowed to cancel when reducing fractions?
5. What do you do with the LCD when you are solving an equation?

You multiply each of the terms on both sides of the equation by the LCD to eliminate the denominators in the equation.
Perform the indicated operations.

\[
\frac{7x}{2x^3} \div \frac{2x^2}{5}
\]
Perform the indicated operations.

\[
\frac{5}{4x} + \frac{7}{12}
\]
\[
\frac{x}{4} + \frac{1}{2} = \frac{7}{8}
\]

Solve the equation
\[
3 - \frac{4x + 2}{3} = \frac{x - 1}{6}
\]

Solve the equation
8. Simplify. Write with positive exponents only.

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^5x^3$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(3x^4)^2$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{4x}{2x^3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. Simplify. Write with positive exponents only.

<table>
<thead>
<tr>
<th>Expression</th>
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<th>Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4x^{-3}$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8. Simplify. Write with positive exponents only.

\[
\frac{\left(-2x^2\right)^4}{3x^{-3}}
\]
9. Use calculator to evaluate:
\[(8.6 \times 10^{15})(3.4 \times 10^{11})\]
9b. Use a Calculator

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-8.4^4)</td>
<td></td>
</tr>
<tr>
<td>((-8.4)^4)</td>
<td></td>
</tr>
</tbody>
</table>
### 9b. Which one is different?

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$5^{-2} =$</td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{5^2} =$</td>
<td></td>
</tr>
<tr>
<td>$-5^2 =$</td>
<td></td>
</tr>
<tr>
<td>$\left(\frac{1}{5}\right)^2 =$</td>
<td></td>
</tr>
</tbody>
</table>
### 11. Compute

<table>
<thead>
<tr>
<th>Expression</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-3^2 = $</td>
<td></td>
</tr>
<tr>
<td>$(-3)^2 = $</td>
<td></td>
</tr>
<tr>
<td>$-3^{-2} = $</td>
<td></td>
</tr>
<tr>
<td>$(3)^{-2} = $</td>
<td></td>
</tr>
</tbody>
</table>
12. Write the number in Scientific Notation

A number in Scientific Notation has the form $P \times 10^n$ where $1 \leq P < 10$ and $n$ is an integer.

$36,000,000 = \boxed{3.6 \times 10^7}$
13. Write the number in Decimal Notation

A number in Scientific Notation has the form $P \times 10^n$ where $1 < P < 10$ and $n$ is an integer.

$2.4 \times 10^{-3}$
14a. There are 200 bacteria initially present in a culture. The culture grows at a rate of 4% a day. Complete the table.

<table>
<thead>
<tr>
<th>Time</th>
<th>Calculation</th>
<th># of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial Day</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 Day Later</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 Days Later</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 Days Later</td>
<td></td>
<td></td>
</tr>
<tr>
<td>n Days Later</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
14b. Use the equation to find the number of bacteria there will be in 25 days.

<table>
<thead>
<tr>
<th>Time</th>
<th>Calculation</th>
<th># of Bacteria</th>
</tr>
</thead>
<tbody>
<tr>
<td>n Days Later</td>
<td>$200 \times (1.04)^n$</td>
<td>$200 \times (1.04)^n$</td>
</tr>
<tr>
<td>25 days later</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
15. Find Ed Rendell’s monthly payments on a 36-month car loan of $10,000 at 6% annual interest.
16. A mathematics professor scales her exam so that $7/8$ of the class passes. If 24 students are in the class, how many will pass?
\[
\begin{array}{c|c}
\text{Circle the mistake!} & \\
\hline
1 - \frac{3x - 1}{3} = \frac{2 - x}{6} & \\
6(1) - 6 \left( \frac{3x - 1}{3} \right) = 6 \left( \frac{2 - x}{6} \right) & \\
6 - 2(3x - 1) = 2 - x & \\
6 - 6x - 2 = 2 - x \quad (-2)(-1) = +2 & \\
-6x + 4 = 2 - x & \\
4 = 2 + 5x & \\
2 = 5x & \\
\frac{2}{5} = x & \\
\end{array}
\]