Review Conics
9.1 Sequences and Series
Sequences
Factorial Notation
Summation Notation

Worksheet on Conic Sections

Homework: 9.1

Quiz on Conics
(a) \[ \frac{x^2}{144} + \frac{y^2}{169} = 1 \]

\[ a = 13 \]
\[ b = 12 \]
Find the center and vertices, identify the graph of the equation and sketch the graph:

\[
\frac{x^2}{16} - \frac{y^2}{25} = 1
\]

Identify which conic section: Hyperbola

Characteristics applicable:
- Center: (0, 0)
- Vertices: (-4, 0) and (4, 0)
- Asymptotes: \(y = \pm \frac{5}{4}x\)

\[
\begin{align*}
\text{Asymptotes:} & \quad y = \frac{5}{4}x \\
& \quad y = -\frac{5}{4}x
\end{align*}
\]
Complete the square, identify the graph of the equation, identify the characteristics of the conic section and graph: 

$$9x^2 + 25y^2 - 36x - 50y + 25 = 0$$

Completed square:

$$9(x^2 - 4x + 4) + 25(y^2 - 2y + 1) = 164 + 36 + 25$$

$$9(x - 2)^2 + 25(y - 1)^2 = \frac{225}{225}$$

$$\frac{(x - 2)^2}{25} + \frac{(y - 1)^2}{9} = 1$$
Complete the square, identify the graph of the equation, identify the characteristics of the conic section and graph: \( 9x^2 + 25y^2 - 36x - 50y + 286 = 0 \)

\[
\frac{(x-2)^2}{5^2} + \frac{(y-1)^2}{3^2} = 1
\]

Identify: Ellipse
Center: (2,1)
Major: (-3,1) (7,1)
Minor: (2,4) (2,-2)

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\[ \frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{4} = 1 \]
Examples: Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola; find the center and vertices.

1. \[ x^2 + 4y^2 - 6x + 16y + 21 = 0 \]
2. \( y^2 - 4y - 4x = 0 \)

\[ y^2 - 4y + 4 = 4x + 4 \]

\[ (y - 2)^2 = 4(x + 1) \]

\[ (y - 2)^2 = 4(x - (-1)) \]

\[ (x + 1)^2 = 4(y - 2) \]

\[ 4p = 4 \]

\[ p = 1 \]

\[ y - 2 = \pm 2 \sqrt{x + 1} \]

\[ y - 2 = \pm 2 \]

\[ y = 0, y = 4 \]
3. \[ 4y^2 - 2x^2 - 4y - 8x - 15 = 0 \]

\[
4y^2 - 4y + \underbrace{-2x^2 - 8x}_{15} = 15 + 1 - 8
\]

\[
4(y^2 - y + \frac{1}{4}) - 2(x^2 + 4x + 4) = 15 + 1 - 8
\]

\[
4(y - \frac{1}{2})^2 - 2(x + 2)^2 = \frac{8}{8}
\]

\[
\frac{(y - \frac{1}{2})^2}{2} - \frac{(x + 2)^2}{4} = 1
\]

Hyperbola

Center \((-2, \frac{1}{2})\)
\[
\frac{(y - \frac{1}{2})^2}{\frac{1}{4}} - \frac{(x + 2)^2}{4} = 1
\]
\[
\frac{(y - \frac{1}{2})^2}{2} - \frac{(x+2)^2}{4} = 1
\]

Hyperbola

Center \((-2, \frac{1}{2})\)

Vertices \((-2, \frac{1}{2} + \sqrt{2})\), \((-2, \frac{1}{2} - \sqrt{2})\)

\[
\begin{align*}
(y - \frac{1}{2}) & = \frac{\sqrt{2}}{2} (x + 2) \\
\frac{y}{a} & = \frac{x + 2}{\sqrt{2}} \\
\frac{y - \frac{1}{2}}{a} & = -\frac{\sqrt{2}}{2} (x + 2) \\
y & = \frac{1}{2} + \frac{\sqrt{2}}{2} (x + 2)
\end{align*}
\]
Definitions:
An **infinite sequence** is a function whose domain is the set of positive integers.
The function values \(a_1,a_2,\ldots,a_n,\ldots\) are the **terms** of the sequence.
If the domain of the function consists of the first \(n\) positive integers only, the sequence is a **finite sequence**.
The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

where $i$ is called the index of summation, $n$ is the upper limit of summation, and 1 is the lower limit of summation. This is called a finite series.
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If $n = \infty$, the series is called an **infinite series**.
is the Greek letter sigma
\[ \sum_{i=1}^{n} x_i \] is called sigma notation or summation notation.
Instead of writing $f(n)$, we write $a_n$. 
Write the first four terms of the following sequences
1. \( a_n = 2n + 5 \)
\[ a_1 = 2(1) + 5 = 7 \]
\[ a_2 = 2(2) + 5 = 9 \]
\[ a_3 = 2(3) + 5 = 11 \]
\[ a_4 = 2(4) + 5 = 13 \]
2. $b_n = 3^{n-1}$
\[ b_1 = 3^1 - 1 = 2 \]
\[ b_2 = 3^2 - 1 = 8 \]
\[ b_3 = 3^3 - 1 = 26 \]
\[ b_4 = 3^4 - 1 = 80 \]
$c_n = \frac{(-1)^n}{n^2 + 1}$
\[ c_1 = \frac{(-1)^1}{1^2 + 1} = -\frac{1}{2} \]
\[ c_2 = \frac{(-1)^2}{2^2 + 1} = \frac{1}{5} \]
\[ c_3 = \frac{(-1)^3}{3^2 + 1} = -\frac{1}{10} \]
\[ c_4 = \frac{(-1)^4}{4^2 + 1} = \frac{1}{17} \]
If $n$ is a positive integer, $n$ factorial is defined as
$$n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n.$$  
As a special case, zero factorial is defined by $0! = 1$. 
What is the difference between $2n!$ and $(2n)!$?
\[ 2n! = 2n(n-1)(n-2) \cdots 2 \cdot 1 \]

\[ (2n)! = 2n(2n-1)(2n-2) \cdots 2 \cdot 1 \]
Write the first four terms of the following sequence.

\[ a_n = \frac{n^2}{n!} \]
\[ a_1 = \frac{1^2}{1!} = 1, \quad a_2 = \frac{2^2}{2!} = 2, \quad a_3 = \frac{3^2}{3!} = \frac{3}{2}, \quad a_4 = \frac{4^2}{4!} = \frac{2}{3} \]
Evaluate the factorial expressions.
\[
\frac{10!}{2! \cdot 8!}
\]
\[
\frac{10!}{2! \cdot 8!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 8!} = \frac{10 \cdot 9}{2} = 5 \cdot 9 = 45
\]
b) \( \frac{n!}{(n+1)!} \)
\[
\frac{n!}{(n+1)!} = \frac{n!}{(n+1)n!} = \frac{1}{n+1}
\]
We will be concerned only with finite series.
1. \[ \sum_{i=1}^{n} c = cn, \] \( c \) a constant.
2. $\sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i$, $c$ a constant.
3. \[ \sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \]
4. \[ \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \]
Examples: Use sigma notation to write the sum.

1. \( \frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15} \)
2. \(-\frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \cdots - \frac{1}{128}\)
3. \[ \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{10 \cdot 12} \]
Examples: Find the sum.

4. \[ \sum_{i=1}^{6} (3i - 1) \]
5. \[ \sum_{i=1}^{5} 6 \]
6. \[ \sum_{k=2}^{5} (k + 1)(k - 3) \]