Review Questions Chapter 3
Asymptotes Worksheets

**Homework:** Review Chapter 3

Test on Chapter 3 Wednesday
Mat 161 Topics for Test 3

- Complete the square, identify vertex, y-intercept, x-intercepts, sketch graph.
- Given graph and two points, find the function.
- Determine the left and right hand behavior of graph.
- Use synthetic division to divide.
- Match graph with function.
- List possible rational zeros.
- Find all real solutions.
- Find all solutions.
- Find polynomial given the zeros.
- Find all linear factors.
- Identify horizontal and vertical asymptotes
- Identify slant asymptotes
- Application problems
3. Find the equation of the quadratic that goes through the following points.
2. The cost function is \( C = 0.8x^2 - 17x + 300 \). The revenue function is \( R = -0.2x^2 + 39x \). Find when the profit is a maximum.
1. A rectangular box with a square base is to be formed from a square piece of metal with 12-inch sides. A square piece with side $x$ is cut from the corners of the metal and the sides are folded up to form an open box. What value of $x$ will maximize the volume of the box?
1. Find the minimum pt on the graph.

\[ y = 2x^2 + 8x + 9 \]
2. Find a quadratic that has a maximum at (-1, 2) and passes through the point (0, 1).

\[ f(x) = a(x-h)^2 + k \]

Given that the maximum is at (-1, 2), we have:

\[ f(-1) = 2 \]

Substituting the point (0, 1) into the equation:

\[ f(0) = a(0+1)^2 + 2 \]

Solving for \( a \):

\[ 1 = a + 2 \]

\[ a = -1 \]
\[ P = -0.0002x^2 + 140x - 250,000 \]

\[ x = \frac{-b}{2a} = \frac{-140}{2(-0.0002)} = 350,000 \text{ units} \]

\[ P(350,000) = -0.0002(350,000)^2 + 140(350,000) - 250,000 \]
4. Find all real zeros of the polynomial.

\[ g(t) = t^3 + 3t^2 - 16t - 48 \]
5. Find a $3^{rd}$ polynomial function with zeros 0, 1, -2.
#6 [0,1]

\[ f(x) = 2x^3 + 7x^2 - 1 \]

\[ f(0) = -1 \]

\[ f(1) = 2 + 7 - 1 = 8 \]

\[ f(0.36) = 5.12 \times 10^{-4} \]

\[ f(0.35) = -0.056 \]

\[ f(0.36) = 0.000512 \]

\[ f(0.3) = -0.316 \]

\[ f(0.4) = 0.248 \]
7. Use the Rational Zero Test to list all possible rational zeros of f. Use a graphing utility to verify that the zeros of f are contained in your list.

\[ f(x) = 2x^4 - 17x^3 + 35x^2 + 9x - 45 \]
8. Find all real zeros: \( x^3 + 8x^2 + 17x + 6 = 0 \)
\[ x = 2 + i \quad x = 2 - i \]
\[ (x - 2 - i)(x - 2 + i) \]

Zeroes: 1, -1, i, \text{ 4th degree polynomial with real coefficients}
10. Given \( f(x) = x^4 - 16 \)

   a. Write as a product of factors irreducible over rationals.

   b. Write in completely factored form.

\[
x^4 - 16 = (x^2 + 4)(x^2 - 4)
\]

\[
(x^2 + 4)(x+2)(x-2)
\]

\[
= (x+2i)(x-2i)(x+2)(x-2)
\]

(complex - completely factored form)
\[ x^4 - 9 = (x^2 + 3)(x^2 - 3) \]

\[ (x^2 + 3)(x + \sqrt{3})(x - \sqrt{3}) \]

\[ (x + i\sqrt{3})(x - i\sqrt{3})(x + \sqrt{3})(x - \sqrt{3}) \]
11. Find the vertical asymptote(s) for \( \frac{x+5}{x^2+4} = 0 \)

y-intercept: \((0, \frac{5}{4})\)

HA: \(y = 0\)

VA: none

x-intercept: \(x = -5\)

\(1 + \frac{5}{x^2} \to 0\)

\(x \to \infty\)

\(y \to 1\)

\(x^2 + 4 = 0\)

\(x^2 = -4 = 4i^2\)

\(x = \pm 2i\)
\[ y = x + 2 \]

\[ 0 = x + 2 \quad \Rightarrow \quad x = -2 \]

\[ y = \frac{x + 2}{x - 5} \]

\[ 0 = \frac{x + 2}{x - 5} \quad (-2, 0) \]
12. Find the vertical asymptote(s) for \( \frac{x + 5}{x^2 - 4} \)
13. Find the vertical asymptote for

\[ f(x) = x + 2 - \frac{3}{x} \]
14. Find horizontal asymptote for \( f(x) = \frac{7}{x - 4} \)

- **x-intercept:** None
- **H A:** \( y = 0 \)
- **y-intercept:** \( (0, \frac{7}{4}) \)
- **V A:** \( x = 4 \)

\[
\lim_{{x \to \infty}} \frac{7}{x - 4} = 0
\]

\[
x \to \infty, \quad y \to 0
\]
15. Find horizontal asymptote for \( f(x) = \frac{7x}{x-4} \)

\( H A : y = 7 \)

\( V A : x = 4 \)

x-intercept: \((0, 0)\)

y-intercept: \((0, 0)\)

\[
\frac{7}{1 - \left(\frac{4}{x}\right)} \rightarrow \frac{0}{0}
\]

\( x \rightarrow \infty \)

\( y > 7 \)
16. Find the slant asymptote for:

\[ f(x) = \frac{x^2 + 3x + 1}{x + 1} \]

\[ \begin{array}{c}
\frac{x + 2}{x + 1} \\
\frac{x^2 + 3x + 1}{x + 1} \\
\frac{x^2 + 1x}{2x + 1} \\
\frac{2x + 1}{2x + 2} \\
\frac{y}{-1}
\end{array} \]

VA: \( x = -1 \)

SA: \( y = x + 2 \)

y-intercept: \( (0, 1) \)
3. \[ g(x) = \frac{3x^4 - 5x + 3}{x^4 + 1} \]
2. \[ f(x) = \frac{x + 4}{x^2 + x - 6} \]
Find the horizontal asymptote(s).

\[ f(x) = \frac{3x^2 + 2x - 16}{x^2 - 7} \]
Find domain \( f(x) = \frac{3x - 1}{x^2 + 9} \)
Find the x-intercepts \( f(x) = \frac{x + 2}{x - 1} \)
5. Determine the vertical asymptote(s) for

\[ f(x) = \frac{x}{x + 1} \]
1. Find any horizontal and vertical asymptotes for

\[ f(x) = \frac{x}{x^3 - 1} \]
2. Graph \( f(x) = \frac{x + 6}{x + 2} \)

a. Find vertical asymptote(s)  

b. Find horizontal asymptote(s)  

c. \( x \) intercept(s)  

d. \( y \) intercept(s)
3. Find the slant asymptote for 

\[ f(x) = \frac{x^2 + 2x - 1}{x - 1} \]

4. Sketch 

\[ f(x) = \frac{x^2 + 2x - 1}{x - 1} \]
List the asymptotes and graph:

\[ f(x) = \frac{3x^2 + x - 5}{x^2 + 1} = 3 + \frac{x - 8}{x^2 + 1} \]

Draw a sign chart for the rational part of the function!
4. \[ f(x) = \frac{x^2 + 5x + 8}{x + 3} \]
Find the function of the following graphs. The Window is:

- X min = -6  Y min = -8
- X max = 6   Y max = 8
- X scl = 2   Y scl = 2

1. 4th degree polynomial
2. 4th degree polynomial
3. 3rd degree polynomial
4. What are the right-hand and left-hand behavior of
   a. \( f(x) = 5x^3 + 7x^2 - 2x + 6 \)
   
   b. \( g(x) = -3x^4 + 6x^2 + 4x - 10 \)
5. Factor: $5x^3 - 28x^2 - 11x - 6$
6. Find the zeros of the polynomial $f(x) = x^4 - 8x^2 + 72x - 65$
7. Is the zero of even or odd multiplicity?
The cost of producing $x$ units is $C = 0.25x^2 + 5x + 78$. The average cost per unit is

$$\bar{C} = \frac{0.25x^2 + 5x + 78}{x} = 0.25x + 5 + \frac{78}{x}.$$

Find the number of units that should be produced to minimize the average cost. Graph this function on a graphing utility, then use the “minimum” command. $x \approx 17.66$
A game commission has determined that if 500 deer are introduced into a preserve, the population at any time $t$ (in months) is given by

$$N = \frac{500 + 350t}{1 + 0.2t}.$$ What is the carrying capacity of the preserve?

The carrying capacity will be the horizontal asymptote, $y = 1750$. 
In a pilot project, a rural township is given recycling bins for separating and storing recyclable products. The cost $C$ (in dollars) for supplying bins to $p\%$ of the population is given by

$$C = \frac{25,000p}{100-p}, 0 \leq p < 100.$$  

(a) Find the cost of supplying bins to 15% of the population.

(b) Find the cost of supplying bins to 50% of the population.

(c) Find the cost of supplying bins to 90% of the population.
(d) Use a graphing utility to graph the cost function. Be sure to choose an appropriate viewing window. Explain why you chose the values that you used in your viewing window.

(e) According to this model, would it be possible to supply bins to 100% of the residents? Explain.
Example 2. The following table gives the mileage $y$, in miles per gallon, of a certain car at various speeds $x$ (in miles per hour).

a) Use a graphing utility to create a scatter plot of the data.
b) Use the regression feature of a graphing utility to find a quadratic model that best fits the data.
c) Use the model to predict the speed that gives the greatest mileage.

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<th>Speed, $x$</th>
<th>Mileage, $y$</th>
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<tr>
<td>10</td>
<td>21.3</td>
</tr>
<tr>
<td>15</td>
<td>23.7</td>
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