Mat 161 Day 10 Agenda 9/23/05

Return Quiz

Quick Review Sheet
Application Problems
Questions
Review sheets for test 1

Monday
Test 1: Chapter 1 and Preliminary Chapter
#3

\[(x-h)^2 + (y-k)^2 = r^2\]

- Center \((-h, k)\)
- Radius \(r\)

- \((x-1)^2 + (y-2)^2 = 9\)
- Radius \(3\)
\[(x-2)^2 + (y+1)^2 = r^2\]

\[r = 4\]

\[\text{standard form}\]

\[(x-2)^2 + (y+1)^2 = 16\]

\[x^2 - 4x + 4 + y^2 + 2y + 1 = 16\]

\[(x-2)^2(x-2)^2\]

\[x^2 - 2x - 2x + 4\]
\[ \alpha^5 - 3 \alpha^2 = \alpha^2 (\alpha - 3) \]

\[ 2\alpha (\alpha - 5)^{-3} - 4 \alpha^2 (\alpha - 5)^{-4} \]

\[ \frac{\alpha^5 - 3\alpha^2}{\alpha^2 (\alpha - 3)} \]

\[ 2\alpha (\alpha - 5)^{-4} \left[ (\alpha - 5)^{\frac{-3+4}{-3-(-4)}} - 2\alpha \right] \]

\[ 2\alpha (\alpha - 5)^{-4} (\alpha - 5 - 2\alpha) \]

\[ 2\alpha (\alpha - 5)^{-4} (-\alpha - 5) = \frac{2\alpha (-\alpha - 5)}{(\alpha - 5)^4} \]
\[
\frac{\text{new} - \text{old}}{\text{old}} = \frac{882 - 939}{939} = -\frac{57}{939} = -0.0607 = -6.07\%
\]
\[ d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} \]

\[ = \sqrt{(-1 - 5)^2 + (2 - 4)^2} \]

\[ = \sqrt{(-6)^2 + (-2)^2} \]

\[ = \sqrt{36 + 4} = \sqrt{40} \]

\[ \approx \sqrt{4 \cdot 10} = 2 \sqrt{10} \]
$y = x^3$

Plot:

- $Y_1 = x^3$
- $Y_2 = -Y_1(x)$
- $Y_3 = -Y_1(x-2)$
- $Y_4 = -Y_1(x-2)+1$

Window:

- $X_{min} = -4.7$
- $X_{max} = 4.7$
- $X_{scale} = 1$
- $Y_{min} = -3.1$
- $Y_{max} = 9.3$
- $Y_{scale} = 1$
- $X_{res} = 1$
1. If \( f(x) = x^2 - 12x \), find \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \)
\[ f(x) = 7x - 2x^2 \]

\[ f(x+h) = 7(x+h) - 2(x+h)^2 \]

\[ = 7x + 7h - 2x^2 - 4xh - 2h^2 \]

\[ f(x+h) - f(x) = 7h - 4xh - 2h^2 \]

\[ \frac{f(x+h) - f(x)}{h} = \frac{7h - 4xh - 2h^2}{h} \quad h \neq 0 \]

\[ \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = 7 - 4x - 2h \]

\[ DQ = 7 - 4x - 2h \]
3. Given \( f(x) = x^3 \) and \( g(x) = 2x + 1 \)

a. Find \((g \circ f)(x) = \)

\[
g(f(x)) = 2x^3 + 1
\]

\[
fog(x) = f(g(x)) = f(2x+1) = (2x+1)^3
\]

b. What is the domain?

________________________________________________________________________

c. Find \((g + f)(x) = \)

\[
x^3 + 2x + 1
\]

d. Find \(g^{-1}(x) = \)

________________________________________________________________________
\[ f(x) = x^3 \]

\[ g \circ f(x) = g(f(x)) = \begin{cases} 
q(x) = 2x + 1 \\
q(x^3) = 2(x^3) + 1 \\
\end{cases} \]

\[ h(x) = 4x - 9 \]

\[ h(-2) = 4(-2) - 9 = -17 \]
\[
\begin{align*}
&\left(\chi^2 + 1\right)^{-2} - \chi^2 \left(\chi^2 + 1\right)^{-3} \\
\Rightarrow \\
&\left(\chi^2 + 1\right)^{-3} \begin{bmatrix} \chi^2 \\ \chi^2 + 1 \end{bmatrix} = \frac{1}{\left(\chi^2 + 1\right)^3}
\end{align*}
\]
\((-\infty, -2)\) \text{inc} \\
\((-2, 0)\) \text{dec} \\
(0, \infty)\text{inc}
#8

\[ f(x) = 2x^3 + 3x^2 \]

\[ f(-x) = 2(-x)^3 + 3(-x)^2 \]

\[ = -2x^3 + 3x^2 \]

\[ -f(x) = -2x^3 - 3x^2 \]
\[ f(x) = f(-x) \quad \text{even} \]

\[ f(x) = 3x^2 - 6 \]

\[ f(-x) = 3(-x)^2 - 6 = 3x^2 - 6 \]
\[ g(x) = -x^2 + 2 \]

\[ f(x) = x^2 \]

\[ -f(x) = -x^2 \]

\[ 2 - f(x) = -x^2 + 2 \]
\[
\sqrt[12]{75x^2y^5} = \sqrt{3 \cdot 25x^2y^4 \cdot \sqrt{y}}
\]

\[
= \sqrt{3 \cdot y \cdot 25x^2y^4}
\]

\[
= 5xy^2 \sqrt{3y}
\]
\[ 3 \sqrt[3]{-\chi^3 \eta^7} = -3 \sqrt[3]{\chi^3 \eta^4 \eta} \]
\[= -3 \sqrt[3]{\chi^3 \eta^4} \sqrt[3]{\eta} \]
\[= -\chi \eta \sqrt[3]{\eta} \]
\[3 \sqrt[3]{-8} = -2\]
\[ \sqrt{25n^2} = |5n| = 5|n| \]
\(\sqrt[6]{8x^3y^3} = (2xy)^{3/6}\)

\(\sqrt{25} = 25^{1/2}\)

\((x^4)^3 = x^{12}\sqrt{2xy}\)
\[
\frac{3}{(\sqrt{7}+2)} \cdot \frac{\sqrt{7} - 2}{(\sqrt{7} - 2)} = \frac{3(\sqrt{7} - 2)}{\sqrt{7} - 2}
\]

\[
\frac{7 - 2\sqrt{7} + 2\sqrt{7} - 4}{\sqrt{7} - 2} = 3
\]
\[ f(x) = x^3 - 3x^2 + 2 \]

\[ \text{max: } f(0), 0 \]
\[ (2, \infty) \]
\[ \text{dec: } (0, 2) \]
\[ (2, -2) \]
\( g(x) = 2x + 1 \)

Let \( y = x \)

\[
\begin{align*}
x &= 2y + 1 \\
\frac{x-1}{2} &= y
\end{align*}
\]

\[
y = \frac{x-1}{2} = g^{-1}(x)
\]

\[
\frac{1}{2}(x-1) = \frac{1}{2}x - \frac{1}{2}
\]
Find the inverse:

48. \( q(x) = (x - 5)^2, x \leq 5 \)
\[ q(x) = (x - 5)^2 \quad x \leq 5 \quad \text{range } y \geq 0 \]

\[ y = (x - 5)^2 \]

Let \( y = x \)

\[ x = (y - 5)^2 \]

\[ \sqrt{x} = \sqrt{(y - 5)^2} \]

\[ \sqrt{x} = |y - 5| \]

\[ -\sqrt{x} = (y - 5) \]

\[ y = 5 - \sqrt{x} \]

\[ q^{-1}(x) = 5 - \sqrt{x} \]
A student's salary in dollars for collecting phone books is given by $S(x) = 30x + 240$
The amount of withholding for taxes is $W(x) = .20x$, where $x$ is salary. Express withholding as a function of number of phone books collected.
An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side, by cutting out equal squares from the corners and turning up the sides.
Find the function for Volume of the box.
A company produces a toy for which the variable cost is $12.30 per unit and the fixed costs are $98,000. The toy sells for $17.98. Let $x$ be the number of units produced and sold.
Page 55 #22. The table shows the number y of Wal-Mart stores for each year x from 1994 through 2001. Sketch a scatter plot of the data.

<table>
<thead>
<tr>
<th>Year, x</th>
<th>Number of stores, y</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>2759</td>
</tr>
<tr>
<td>1995</td>
<td>2943</td>
</tr>
<tr>
<td>1996</td>
<td>3054</td>
</tr>
<tr>
<td>1997</td>
<td>3406</td>
</tr>
<tr>
<td>1998</td>
<td>3599</td>
</tr>
<tr>
<td>1999</td>
<td>3985</td>
</tr>
<tr>
<td>2000</td>
<td>4189</td>
</tr>
<tr>
<td>2001</td>
<td>4414</td>
</tr>
</tbody>
</table>
\[ 2x(x - 5)^{-3} - 4x^2(x - 5)^{-4} \]
Distance Formula, Midpoint, and Equation of a Circle

Given two points in the coordinate plane: \((x_1, y_1), (x_2, y_2)\)

The **distance** \(d\) between the two points is

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]
Find the distance between (2, -5) and (8, 3).

\[ d = \sqrt{(8 - 2)^2 + (3 - (-5))^2} \]
\[ = \sqrt{6^2 + 8^2} \]
\[ = \sqrt{36 + 64} \]
\[ = \sqrt{100} \]
\[ = 10 \]
The **midpoint** of the line segment joining the two points is

\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

Find the midpoint of the line segment joining the points

\((-9, 5) \text{ and } (4, 2)\).

\[
\left( \frac{-9 + 4}{2}, \frac{5 + 2}{2} \right) = \left( -\frac{5}{2}, \frac{7}{2} \right)
\]
Find the standard form of the equation of the circle with center at \((2, -5)\) and radius 4.

\[(x - 2)^2 + (y - (-5))^2 = 4^2\]

or

\[(x - 2)^2 + (y + 5)^2 = 16\]
1. Graph $y = -|x| + 6$
2. Find an equation of a line passing through (-1,-3) and parallel to the line $2x + y = 19$
3. Given $f(x) = \begin{cases} \sqrt{-x}, & x \leq 0 \\ 6x, & x > 0 \end{cases}$ find $f(4)$
4. If \( f(x) = x^2 - 2x \), find \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \)
5. Find the domain of $f(x) = \sqrt{5 - x}$
6. Graph $f(x) = [[x+1]]$
7. Determine where function is increasing, decreasing, and whether it has a relative maximum or relative minimum.
8. Determine whether the functions are odd or even or neither.

a. $f(x) = 2x^3 + 3x^2$

b. $f(x) = 3x^2 - 6$
9. Find an equation of a function that shifts $f(x) = x^2$ two units up vertically, three units to the right horizontally.
10. Graph $g(x) = -x^2 + 2$ using $f(x) = x^2$
11. Given \( f(x) = x + 1 \) \( g(x) = x + 9 \)

a. Find \((g \circ f)(x) = \)

b. Find \((g + f)(x) = \)

c. Find \( f^{-1}(x) = \)
12. Show \( f(x) = \frac{x}{2} \) and \( g(x) = 2x \) are inverse of each other.
<table>
<thead>
<tr>
<th></th>
<th></th>
<th>-4</th>
<th>-2</th>
<th>-2</th>
</tr>
</thead>
</table>

Title: Sep 22 - 5:53 PM (48 of 82)
2. Use inequality notation to describe

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>all real numbers less than 3</td>
</tr>
<tr>
<td>b.</td>
<td>set of real numbers that are less than 4 and at least –2</td>
</tr>
</tbody>
</table>
3. a. Find the distance between –43 and 16.

   b. Use the absolute notation to describe the distance between \( x \) and 16 is no more than 5.
4. What is the degree of $12x^3 + 5x^2 + 2$. 
5. Evaluate $6x^2 - 2x$ for $x = 3$. 
6. Identify the property

\[
5(\frac{1}{5} + x) = 5(\frac{1}{5}) + 5x
\]

\[
3 + (2 + 7) = (3 + 2) + 7
\]

\[
(7 \times 2) \times 4 = 4 \times (7 \times 2)
\]
7. Evaluate $-2x^0 y^2$ for $y = 2, x = 1$
8. Evaluate \( \frac{4(2)^{-1}}{3^{-2}2} \)
9. Simplify \( \left( \frac{x^{-5}y^2}{z^2} \right)^{-3} \)
10. Simplify

a. \((-3x^2)^2 (-3x^2)^2 (-3x^2)^4\)
b. \[\left(\frac{1}{64}\right)^{3/2}\]
11. Simplify and write answers with no exponents.

\[ -\frac{3y^{-2}}{(2y)^{-3}} \]
Examples: Factor completely.

\[ x^2 - x - 6 \]

\[ 2x^2 - 3x - 14 \]

\[ 6x^2 - 29x + 35 \]

To see a review of quadratics factoring, go to the following site:

http://faculty.mc3.edu/rhofman/Flash03/Dec17/menufactor.html
P.4 Rational Expressions

Definitions:
The set of real numbers for which an expression is defined is its **domain**. Two expressions are **equivalent** if they yield the same value for all numbers in their domain.

The quotient of two algebraic expressions is a **fractional expression**.
The quotient of two polynomials is a **rational expression**.
Domains:
For a polynomial, the domain is all real numbers.
For expressions that contain radicals with even indices, the radicand must be greater than or equal to zero.
For rational expressions, the denominator cannot be equal to zero.

Examples:

$5x^3 - 2x^2 + 6x - 8$ is a polynomial, the domain is all real numbers.

$\sqrt{x-2}, \ x-2 \geq 0$; the domain is $[2, \infty)$.

$\frac{x}{x-1}, \ x-1 \neq 0$, the domain is $(-\infty, 1) \cup (1, \infty)$. 
Find the domain:

\[2x^2 - 5x - 2\]

\[6x^2 - 9, x > 0\]

\[\frac{x + 1}{2x + 1}\]

\[\sqrt{6 - x}\]
Simplifying Rational Expressions

The key to simplifying rational expressions is that the numerator and denominator should not have any common factors. If the numerator and denominator do have common factors, they can be eliminated by using

$$\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}$$

Note that $c/c = 1$ only if $c \neq 0$. 
Reduce: \( \frac{x^2 - 2x + 1}{x^2 - 1} \)
\[
\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{(x - 1)^2}{(x + 1)(x - 1)} \\
= \frac{x - 1}{x + 1} \cdot \frac{x - 1}{x - 1} \\
= \frac{x - 1}{x + 1}, x \neq 1
\]
Reduce:

\[
\frac{x^2 + 8x - 20}{x^2 + 11x + 10}
\]

\[
\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}
\]
Simplify: \[
\frac{x^3 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1}
\]
\[
\frac{x^2 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} = \frac{(x + 1)(x^2 - x + 1)}{(x - 3)^2} \cdot \frac{(x + 3)(x - 3)}{x^2 - x + 1}
\]
\[
= \frac{(x + 1)(x^2 - x + 1)(x + 3)(x - 3)}{(x - 3)^2(x^2 - x + 1)}
\]
\[
= \frac{(x + 1)(x + 3)}{(x - 3)} \cdot \frac{x^2 - x + 1}{x^2 - x + 1}
\]
\[
= \frac{(x + 1)(x + 3)}{(x - 3)} \cdot 1, x \neq 3
\]
\[
= \frac{(x + 1)(x + 3)}{(x - 3)}, x \neq 3
\]
Simplify: \( \frac{x+6}{x} + \frac{2}{x^2 + x} \)
\[ \frac{x + 6}{x} + \frac{2}{x^2 + x} = \frac{x + 6}{x} \cdot \frac{x + 1}{x + 1} + \frac{2}{x(x + 1)} \]
\[ = \frac{(x^2 + 7x + 6) + 2}{x(x + 1)} \]
\[ = \frac{x^2 + 7x + 8}{x(x + 1)} \]
Complex Fractions:
Fractions that contain fractions in the numerator or denominator are called complex fractions. There are two ways to simplify a complex fraction. Here are examples of each.
Order of Operations:

\[
\frac{1 - \frac{2}{x}}{\frac{5}{x - 1} + \frac{4}{x}} = \frac{\frac{x - 2}{x}}{\frac{9x - 4}{x(x - 1)}}
\]

\[
= \frac{x - 2}{x} \cdot \frac{x(x - 1)}{9x - 4}
\]

\[
= \frac{(x - 2)(x - 1)}{9x - 4}, \quad x \neq 0
\]
Multiply by the LCD.

\[
\frac{1 - \frac{2}{x}}{\frac{5}{x-1} + \frac{4}{x}} = \frac{1 - \frac{2}{x}}{\frac{5}{x-1} + \frac{4}{x}} \cdot \frac{x(x-1)}{x(x-1)}
\]

\[
= \frac{x(x-1) - 2(x-1)}{5x + 4(x-1)}
\]

\[
= \frac{x^2 - 3x + 2}{9x - 4}
\]
Rationalize the numerator: \( \frac{\sqrt{x + 1} - 1}{x} \)
\[
\frac{\sqrt{x+1} - 1}{x} = \frac{\sqrt{x+1} - 1}{x} \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \\
= \frac{\sqrt{x+1}^2 - 1^2}{x(\sqrt{x+1} + 1)} \\
= \frac{x}{x(\sqrt{x+1} + 1)} \\
= \frac{1}{\sqrt{x+1} + 1}, x \neq 0
\]
Perform the operations and simplify:

\[
\frac{4y - 16}{5y + 15} \cdot \frac{2y + 6}{4 - y}
\]
\[
\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8}
\]
\[ \frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{x^2-1} \]
\[
\frac{x - 4}{x} - \frac{4}{4} = \frac{x}{x} - \frac{4}{4} = 1 - 1 = 0
\]
$5x^5 - 3x^\frac{-3}{2}$
\[ 2x(x-5)^{-3} - 4x^2(x-5)^{-4} \]