Mat 161 Day 5 Agenda

Chapter 1.3 & 1.4 and P.4

Function Worksheet #2

DQ
Increasing/Decreasing
Maximum/Minimum

Quiz 1
\# 8 (2.3, -8.5) \quad m = -\frac{5}{2}

\[ y = mx + b \]

\[-8.5 = -\frac{5}{2} (2.3) + b \]

\[-8.5 = -11.5 + b \]

\[-8.5 = -5.75 + b \]

\[ b = -2.75 \]

\[ y = -2.5x - 2.75 \]

\[ 2.5x + y + 2.75 = 0 \]

\[ Ax + By + C = 0 \]
Did you get into BlackBoard?
1.4 Graphs of Functions
- Domain and Range of a Function
- Vertical Line Test
- Increasing and Decreasing Functions
- Relative Minimum and Maximum Values
- Step Functions and Piecewise-Defined Functions
- Even and Odd

P.4 Rational Expressions
- Domain of an Algebraic Expression
- Simplifying Rational Expressions
- Operations with Rational Expressions
- Complex Fractions
- Difference Quotients
Mat 161 Worksheet #2

1. Let \( g(x) = -x^2 + 3x + 1 \)

a) \( g(2) = -(2)^2 + 2(2) + 1 = \)
\[ -4 + 6 + 1 \]
\[ = 3 \]

b) \( g(t) = -t^2 + 3t + 1 + 3 \)
\[ = -t^2 + 3t + 4 \]

\[ \text{(from a)} \]

\[ = \]
\[ -t^2 + 4t + 6 + 1 \]
\[ = -t^2 + 4t + 7 \]

\[ \text{for } g(t+2) \]
\[ = - (t+2)^2 + 3(t+2) + 1 \]
\[ = - \]
\[ -t^2 - 4t - 4 + 3t + 6 + 1 \]
\[ = -t^2 - t + 3 \]

\[ \text{for } g(t+2) \]

\[ \text{Does the answer in a + answer in b = answer in c? that} \]
\[ \text{is: } g(2) + g(t) = g(t+2)? \]

\[ \text{from a) and b:} \]
\[ 3 + -t^2 + 3t + 4 = -t^2 + 4t + 7 \]

\[ \Rightarrow g(t+2) = -t^2 - t + 3 \]
2. Given: \( f(x) = \begin{cases} x^2 - 1 & x < 0 \\ x + 1 & x \geq 0 \end{cases} \) find:

a) \( f(-1) \)  \hspace{1cm} (-1)^2 - 1 = 1 - 1 = 0

b) \( f(0) \) \hspace{1cm} 0 + 1 = 1

c) \( f(1) \) \hspace{1cm} 1 + 1 = 2
\[ \frac{x^2 - 9}{x + 3} = \frac{(x - 3)(x + 3)}{x + 3} = x - 3 \]

\[ \text{for } x \neq -3 \]
Definition:

The ratio \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \) is called the difference quotient.
Example:

a) Find the difference quotient for the function \( f(x) = 5x - x^2 \) for \( x = 5 \).

b) 
\[
\frac{f(x + h) - f(x)}{h} = \frac{[(x + h)^2 + (x + h) - 1] - [x^2 + x - 1]}{h}
\]
\[
= \frac{x^2 + 2xh + h^2 + x + h - 1 - x^2 - x + 1}{h}
\]
\[
= \frac{2xh + h^2 + h}{h}
\]
\[
= 2x + h + 1
\]
$f(x) = 5x - x^2$

\[
DQ = \frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - (x^2)}{(x+h)(x+h)}
\]

\[
f(x) = 5x - x^2
\]

\[
f(x+h) = 5(x+h) - (x+h)^2
\]

\[
= 5x + 5h - (x^2 + 2xh + h^2)
\]

\[
f(x+h) = 5x + 5h - x^2 - 2xh - h^2
\]

\[
- f(x) = -5x + x^2
\]

\[
f(x+h) - f(x) = 5h - 2xh - h^2
\]

\[
\frac{f(x+h) - f(x)}{h} = \frac{5h - 2xh - h^2}{h} = \frac{h(5 - 2x - h)}{h} \quad h \neq 0
\]

\[
DQ = \frac{f(x+h) - f(x)}{h} = 5 - 2x - h \quad h \neq 0
\]
To see animation of the difference quotient, go to the following site:

http://faculty.mc3.edu/rhofman/Flash03/Web/dq14.htm
Because \( y = f(x) \), graphing *functions* is no different from graphing *equations* in two variables.

**Definitions:** The **graph of a function** \( f \) is the collection of ordered pairs \( (x, y) \) such that \( x \) is in the domain of \( f \).

\[
\begin{align*}
x & = \text{the directed distance from the } y\text{-axis} \\
y & = \text{the directed distance from the } x\text{-axis}
\end{align*}
\]
Definitions:
If \( f \) is defined by an algebraic expression and the domain is not specified, the **implied domain** consists of all real numbers for which the expression is defined.
Example: Find the domain and range

\[ -1 \leq y \leq 3 \]

\[ 1 \leq x \leq 4 \quad [1, 4] \]

\[ -1 \leq x < 4 \quad [-1, 4) \]

\[ -5 \leq y \leq 4 \quad [-5, 4] \]
Examples:
Graph each of the following on your calculator.
Determine the domain and range of each function.
For the last two functions, graph in both connected and dot mode.

\[ y = \sqrt{x + 2} \]

\[ f(x) = \sqrt{x + 2} \]

\[ f(x) = \frac{x + 1}{x + 2} \]

\[ f(x) = \frac{x^2 - 1}{x - 1} \]

\[ x + 2 \geq 0 \]
\[ x \geq -2 \]
\[ [-2, \infty) \]

\[ x \neq -2 \]
\[ (-\infty, -2) \cup (-2, \infty) \]

\[ (x - 1)(x + 1) \]
\[ (x - 1)(x + 1) = x + 1 \quad \text{as } x \neq 1 \]
Find the domain of \( f(x) = \sqrt{x + 1} \).

<table>
<thead>
<tr>
<th>Algebraic Solution</th>
<th>Graphical Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x + 1 \geq 0 )</td>
<td><img src="image" alt="Graph of ( f(x) = \sqrt{x + 1} )" /></td>
</tr>
<tr>
<td>( x \geq -1 )</td>
<td></td>
</tr>
<tr>
<td>([-1, \infty))</td>
<td>([-1, \infty))</td>
</tr>
</tbody>
</table>
Graph: \( f(x) = x^3 + 3x^2 \)

a) Identify any intervals over which you think the function is increasing.
\((-\infty, -2)\) \((0, \infty)\)

b) Decreasing?
\((-\infty, 0)\)
Definitions: A function $f$ is \underline{increasing} on an interval if, for any $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function $f$ is \underline{decreasing} on an interval if, for any $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function $f$ is \underline{constant} on an interval if, for any $x_1$ and $x_2$ in the interval, $f(x_1) = f(x_2)$.
Definitions: A function value $f(a)$ is called a **relative minimum** of $f$ if there exists an interval $(x_1, x_2)$ that contains a such that $x_1 < x < x_2$ implies $f(a) \leq f(x)$.

A function value $f(a)$ is called a **relative maximum** of $f$ if there exists an interval $(x_1, x_2)$ that contains a such that $x_1 < x < x_2$ implies $f(a) \geq f(x)$.

Relative minima and relative maxima are called **relative extrema**.
Example: Use a graphing utility to find any relative minimum or relative maximum values of the function and those intervals where $f$ is increasing and those intervals where $f$ is decreasing.

1. $f(x) = (x - 1)^2 (x + 2)$
2. $g(x) = x^3 - 6x^2 + 15$
3. $h(x) = x \sqrt{4 - x}$
Example: (p.123 #82)

The cost of sending an overnight package from New York to Atlanta is $9.80 for a package weighing up to but not including 1 pound and $2.50 for each additional pound or portion of a pound. Use the greatest integer function to create a model for the cost $C$ of overnight delivery of a package weighing $x$ pounds, where $x > 0$. Sketch the graph of the function.

\[
f(x) = \begin{cases} 
9.80 & 0 \leq x < 1 \\
9.80 + 2.50 \left\lfloor \frac{x}{1} \right\rfloor & x \geq 1
\end{cases}
\]
$y = \lfloor x \rfloor$
\[ C = 9.80 + 2.50 \|x\| \]
**Piecewise defined functions**

Drawing the whole graph for one part of the function on the board, then erase the part that you do not want.

**Example** Sketch the graph of

\[ f(x) = \begin{cases} 
  x + 1, & \text{if } x \leq 2 \\
  1 - x, & \text{if } x > 2 
\end{cases} \]

Draw, then erase, the light parts.
Even and Odd Functions
Even functions are symmetric with respect to the $y$-axis
Odd functions are symmetric with respect to the origin

Definitions: A function $f$ is:
\textbf{even} if, for each $x$ in the domain of $f$, $f(-x) = f(x)$.

\textbf{odd} if, for each $x$ in the domain of $f$, $f(-x) = -f(x)$.

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Symmetric to $y$-axis  Symmetric to origin  Symmetric to $x$-axis

Even  Odd  Not a function
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<tr>
<td>( f(x) = x^4 -</td>
<td>x</td>
</tr>
<tr>
<td>( f(-x) = (-x)^4 -</td>
<td>-x</td>
</tr>
<tr>
<td>( = x^4 -</td>
<td>x</td>
</tr>
<tr>
<td>( = f(x) )</td>
<td></td>
</tr>
<tr>
<td>Therefore the function is even.</td>
<td></td>
</tr>
<tr>
<td>( g(x) = \frac{x}{x^2 + 1} )</td>
<td>Therefore the function is odd.</td>
</tr>
<tr>
<td>( g(-x) = \frac{-x}{(-x)^2 + 1} )</td>
<td></td>
</tr>
<tr>
<td>( = -\frac{x}{x^2 + 1} )</td>
<td></td>
</tr>
<tr>
<td>( = -g(x) )</td>
<td></td>
</tr>
<tr>
<td>Therefore the function is odd.</td>
<td></td>
</tr>
<tr>
<td>( h(x) = x + 6 )</td>
<td>Therefore the function is neither.</td>
</tr>
<tr>
<td>( h(-x) = -x + 6 )</td>
<td></td>
</tr>
<tr>
<td>( \neq h(x), \text{ nor } -h(x) )</td>
<td></td>
</tr>
<tr>
<td>Therefore the function is neither.</td>
<td></td>
</tr>
</tbody>
</table>
Examples: Determine whether the function is even, odd, or neither.

1. \( f(x) = -9 \)

2. \( f(x) = -x^2 - 8 \)

3. \( g(x) = x^3 - 5x \)
Example: Find the coordinates of a second point on the graph of a function $f$ if the given point is on the graph and the function is (a.) even and (b.) odd.

Given point: (-3, 7)
Examples: Factor completely.

\[ x^2 - x - 6 \]

\[ 2x^2 - 3x - 14 \]

\[ 6x^2 - 29x + 35 \]

To see a review of quadratics factoring, go to the following site:

[http://faculty.mc3.edu/rhofman/Flash03/Dec17/menufactor.html](http://faculty.mc3.edu/rhofman/Flash03/Dec17/menufactor.html)
P.4 Rational Expressions

Definitions:
The set of real numbers for which an expression is defined is its domain. Two expressions are equivalent if they yield the same value for all numbers in their domain.

The quotient of two algebraic expressions is a fractional expression.
The quotient of two polynomials is a rational expression.
Domains:
For a polynomial, the domain is all real numbers.
For expressions that contain radicals with even indices, the radicand must be greater than or equal to zero.
For rational expressions, the denominator cannot be equal to zero.

Examples:

$5x^3 - 2x^2 + 6x - 8$ is a polynomial, the domain is all real numbers.

$\sqrt{x - 2}, \ x - 2 \geq 0$; the domain is $[2, \infty)$.

$\frac{x}{x - 1}, \ x - 1 \neq 0$, the domain is $(-\infty, 1) \cup (1, \infty)$. 
Find the domain:

\[ 2x^2 - 5x - 2 \]

\[ 6x^2 - 9, x > 0 \]

\[ \frac{x + 1}{2x + 1} \]

\[ \sqrt{6 - x} \]
Simplifying Rational Expressions

The key to simplifying rational expressions is that the numerator and denominator should not have any common factors. If the numerator and denominator do have common factors, they can be eliminated by using

\[
\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b}
\]

Note that \( c/c = 1 \) only if \( c \neq 0 \).
Reduce: \[
\frac{x^2 - 2x + 1}{x^2 - 1}
\]
\[
\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{(x - 1)^2}{(x + 1)(x - 1)} \\
= \frac{x - 1}{x + 1} \cdot \frac{x - 1}{x - 1} \\
= \frac{x - 1}{x + 1}, x \neq 1
\]
Reduce:

\[
\frac{x^2 + 8x - 20}{x^2 + 11x + 10}
\]

\[
\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}
\]
Simplify: \[ \frac{x^3 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} \]
\[
\frac{x^3 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} = \frac{(x+1)(x^2 - x + 1)}{(x-3)^2} \cdot \frac{(x+3)(x-3)}{x^2 - x + 1}
\]
\[
= \frac{(x+1)(x^2 - x + 1)(x+3)(x-3)}{(x-3)^2(x^2 - x + 1)}
\]
\[
= \frac{(x+1)(x+3)}{(x-3)} \cdot \frac{(x^2 - x + 1)(x-3)}{(x^2 - x + 1)(x-3)}
\]
\[
= \frac{(x+1)(x+3)}{(x-3)}, x \neq 3
\]
Simplify: \( \frac{x + 6}{x} + \frac{2}{x^2 + x} \)
\[
\frac{x + 6}{x} + \frac{2}{x^2 + x} = \frac{x + 6}{x} \cdot \frac{x + 1}{x + 1} + \frac{2}{x(x + 1)}
\]
\[
= \frac{(x^2 + 7x + 6) + 2}{x(x + 1)}
\]
\[
= \frac{x^2 + 7x + 8}{x(x + 1)}
\]
Complex Fractions:
Fractions that contain fractions in the numerator or denominator are called complex fractions. There are two ways to simplify a complex fraction. Here are examples of each.
Order of Operations:

\[
1 - \frac{2}{x} \cdot \frac{5}{x-1} + \frac{4}{x} = \frac{x-2}{x} \cdot \frac{x}{9x-4} \cdot \frac{x(x-1)}{x(x-1)}
\]

\[
= \frac{x-2}{x} \cdot \frac{x(x-1)}{9x-4}
\]

\[
= \frac{(x-2)(x-1)}{9x-4} \cdot \frac{x}{x}
\]

\[
= \frac{(x-2)(x-1)}{9x-4}, \ x \neq 0
\]
Multiply by the LCD.

\[
\frac{1-\frac{2}{x}}{\frac{5}{x-1} + \frac{4}{x}} = \frac{1-\frac{2}{x}}{\frac{5}{x-1} + \frac{4}{x}} \cdot \frac{x(x-1)}{x(x-1)}
\]

\[
= \frac{x(x -1) - 2(x -1)}{5x + 4(x -1)}
\]

\[
= \frac{x^2 - 3x + 2}{9x - 4}
\]
Rationalize the numerator: \[
\frac{\sqrt{x+1}-1}{x}
\]
\[
\frac{\sqrt{x+1} - 1}{x} = \frac{\sqrt{x+1} - 1}{x} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1}
\]

\[
= \frac{\sqrt{x+1}^2 - 1^2}{x(\sqrt{x+1} + 1)}
\]

\[
= \frac{x}{x(\sqrt{x+1} + 1)}
\]

\[
= \frac{1}{\sqrt{x+1} + 1}, x \neq 0
\]
Perform the operations and simplify:

\[
\frac{4y - 16}{5y + 15} \cdot \frac{2y + 6}{4 - y}
\]
\[
\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8}
\]
\[ \frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{x^2-1} \]
\[ \frac{x - 4}{x - \frac{4}{x}} \]
$5x^5 - 3x^{-\frac{3}{2}}$
\[2x(x - 5)^{-3} - 4x^2(x - 5)^{-4}\]