2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 1

Solve $x^2 - x - 12 = 0$

\[(x - 4)(x + 3) = 0\]

$x = 4, -3$
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 2

(a) Solve $2x^2 - 5 = 0$

\[
\frac{2x^2}{2} = \frac{5}{2}
\]

\[
x^2 = \frac{5}{2}
\]

\[
x = \pm \sqrt{\frac{5}{2}}
\]

(b) Solve $(2x - 3)^2 = 7$

\[
2x - 3 = \pm \sqrt{7}
\]

\[
x = 3 \pm \sqrt{7}
\]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 3 (Completing The Square)

Solve \( x^2 - 4x - 10 = 0 \)

\[
\begin{align*}
\underbrace{x - 4 \quad x + 4 = 10 + 4} \\
(x - 2)^2 = 14 \\
\begin{align*}
& x - 2 = \pm\sqrt{14} \\
& x = 2 \pm \sqrt{14}
\end{align*}
\end{align*}
\]
\[ 3x^2 - 6x - 10 = 0 \]

\[ 3 (x^2 - 2x + 1) = 10 + 3 \]

\[ 3 (x-1)^2 = 13 \]

\[ (x-1)^2 = \frac{13}{3} \]

\[ x-1 = \pm \sqrt{\frac{13}{3}} \]

\[ x = 1 \pm \sqrt{\frac{13}{3}} \]

\[ x = 1 \pm \frac{\sqrt{139}}{3} \]

\[ x = \pm \frac{\sqrt{139}}{3} \]
\[2x^2 - 4x - 5 = 0\]

\[2x^2 - 4x - 5 = 5\]

\[2(x^2 - 2x + 1) = 5 + 2\]

\[x^2 - 2x + 1 = \frac{7}{2}\]

\[x - 1 = \pm \sqrt{\frac{7}{2}}\]

\[x = 1 \pm \frac{\sqrt{14}}{2}\]

\[x = \frac{2 \pm \sqrt{14}}{2}\]
\[3x^2 - 2x - 5 = 0\]

Subtract 5 from both sides:

\[3x^2 - 2x = 5\]

Divide by 3:

\[x^2 - \frac{2}{3}x + \frac{1}{3} = 5 + \frac{1}{3}\]

Complete the square:

\[3\left(x - \frac{1}{3}\right)^2 = \frac{16}{9}\]

Take the square root of both sides:

\[x - \frac{1}{3} = \pm \frac{4}{3}\]

Solve for x:

\[x = \frac{1}{3} \pm \frac{4}{3}\]

\[x = \frac{5}{3}, -1\]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 4 (Completing The Square)

Solve \( x^2 - x - 13 = 0 \)

\[
x^2 - x = 13
\]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 5 (Quadratic Formula)

Solve \( x^2 - x - 13 = 0 \)

\[
a x^2 + b x + c = 0
\]

\[
\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

\[
a = 1 \\
b = -1 \\
c = -13
\]

\[
\frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-13)}}{2(1)}
\]

\[
\frac{1 \pm \sqrt{1 + 52}}{2}
\]

\[
\frac{1 \pm \sqrt{53}}{2}
\]

Approximate:

\[
4.14 \\
-3.14
\]

Exact Answer:

\[
\frac{1 \pm \sqrt{53}}{2}
\]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 6 (Quadratic Formula)

Solve \( 4x^2 - 12x + 9 = 0 \)

\[
\begin{align*}
    a &= 4 \\
    b &= -12 \\
    c &= 9
\end{align*}
\]

\[
\frac{-(-12) \pm \sqrt{(-12)^2 - 4(4)(9)}}{2(4)}
\]

\[
12 \pm \sqrt{144 - 144}
\]

\[
\frac{8}{8} \cdot \frac{3}{2}
\]

\[
12/8 = 3/2
\]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 7 (Quadratic Formula)

Solve $3x^2 - 2x + 5 = 0$

$$a = 3$$
$$b = -2$$
$$c = 5$$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(3)(5)}}{2(3)}$$

$$\frac{2 \pm \sqrt{4 - 60}}{6}$$

$$\frac{2 \pm \sqrt{-56}}{6}$$

$$\frac{2 \pm 2i\sqrt{14}}{6}$$

$$x = \frac{1 \pm i\sqrt{14}}{3}$$

$$0.33 + 1.25i$$

$$0.33 - 1.25i$$
2.4 Solving Equations Algebraically – Part 1
Quadratic Type Equations

Example 8 (Quadratic Type)

Solve \( \frac{6}{x} - 5 \left( \frac{x}{x+1} \right) - 6 = 0 \)

\[ x = \frac{s}{s+1} \]

\[ 6x^2 + 5x - 6 = 0 \]

\[ (2x+3)(3x-2) = 0 \]

\[ 2x+3 = 0 \quad 3x-2 = 0 \]

\[ x = -\frac{3}{2} \quad x = \frac{2}{3} \]

\[ \frac{s}{s+1} = -\frac{3}{2} \quad \frac{s}{s+1} = \frac{2}{3} \]

\[ 2s = -3(s+1) \quad 3s = 2(s+1) \]

\[ 2s = -3s - 3 \quad 3s = 2s + 2 \]

\[ 5s = -3 \quad 5s = 2 \]

\[ s = -\frac{3}{5} \quad s = \frac{2}{5} \]

\[ s = -\frac{3}{5} \]

\[ s = 2 \]
2.4 Solving Equations Algebraically – Part 1

Quadratic Type Equations

Example 9 (Quadratic Type)

Solve: \( x^4 - 29x^2 + 100 = 0 \)

\[
(x^2 - 4)(x^2 - 25) = 0
\]

\[
(x + 2)(x - 2)(x + 5)(x - 5) = 0
\]

\( x = \pm 2, \pm 5 \)