1.3 Functions

What is a function?

Example 1

If $500$ is invested at interest rate $r$ compounded quarterly for 2 years, write the account balance $A$ as a function of the interest rate $r$. 

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$
1.3 Functions

Example 2

Determine whether the following equations are functions or not

(a) $x^2 + (y - 3)^2 = 4$

(b) $x^2 - y = 7$

(c) $xy - y - x - 2 = 0$
1.3 Functions

Example 3

For the function \( f(x) = \frac{x + 2}{x - 1} \),

Compute the following:

(a) \( f(2) \)

(b) \( f(-5) \)

(c) \( f(w + 7) \)

(d) \( f(5x^2 + 1) \)

(e) What is the domain of \( f(x) \)?
Example 4

For the function $f(x) = \frac{1}{x}$,

Compute and simplify the difference quotient:

$$\frac{f(x + h) - f(x)}{h}$$
1.3 Functions

Example 5

For the function \( f(x) = x^2 - x - 1 \).

Compute and simplify the difference quotient:

\[
\frac{f(x + h) - f(x)}{h}
\]
\[ f(x) = x^2 - 7x - 1 \]
\[ f(x+h) = (x+h)^2 - 7(x+h) - 1 \]
\[ \frac{(x+h)^2 - 7(x+h) - 1 - (x^2 - 7x - 1)}{h} \]
\[ ax^2 + ax + h^2 - 7h \]
\[ h(2x + h - 7) \]
\[ 2x + h - 7 \]
\[ y = 3x^3 + 7x + 1 \]
\[ 9x^2 + 7x + 0 \]
\[ 1x^0 \]
\[
\frac{1}{x-1} - \frac{1}{x+h-1} = \frac{(x+h-1)(x-1)}{h(x+h-1)(x-1)}
\]

\[
\frac{x-1}{h(x+h-1)(x-1)} - \frac{x - h + 1}{h(x+h-1)(x-1)}
\]

\[
\frac{h}{h(x+h-1)(x-1)}
\]

\[
-\frac{1}{h(x+h-1)(x-1)}
\]
1.3 Functions

Example 6

Find the domain of the given functions.

#58 \( g(x) = 1 - 2x^2 \)
\[ \mathbb{R} \]

#60 \( s(y) = \frac{3y}{y-5} \)
\[ \{ y | y \neq 5 \} \]
\[ \mathbb{R} \]

#62 \( f(t) = \sqrt{t+4} \)
\[ \{ x \} \]
\[ \mathbb{R} \]

#66 \( h(x) = \frac{10}{x^2-2x} \)
\[ \{ x \} \]
\[ \{ x \neq 0, 2 \} \]

#70 \( f(x) = \frac{x-5}{\sqrt{x^2-9}} \)
\[ A(x) = \frac{10}{x^2-2x} \]
\[ x^2-2x = 0 \]
\[ x(x-2) = 0 \]
\[ x = 0 \]
\[ x-2 = 0 \]
\[ x = 2 \]

\[ \sqrt{x^2-9} = 0 \]
\[ x^2-9 = 0 \]
\[ x^2 = 9 \]
\[ x = \pm 3 \]

\[ D = \{ x | x < 3 \text{ or } x > 3 \} \]
1.3 Functions

Example 7 (A Piecewise Function)

For the function \( f(x) = \begin{cases} 
  x^2 - 1 & \text{if } x < 1 \\
  x + 1 & \text{if } x \geq 1
\end{cases} \)

(a) Compute \( f(-3) \), \( f(1) \), \( f(3) \) and \( f(a^2 + 4) \)

\[
\begin{align*}
  f(-3) &= (-3)^2 - 1 \\
        &= 9 - 1 \\
        &= 8 \\

  f(1) &= 1 + 1 \\
       &= 2
\end{align*}
\]

\[
\begin{align*}
  f(3) &= 3 + 1 \\
       &= 4 \\

  f(a^2 + 4) &= \frac{a^2 + 4 + 1}{a^2 + 5} \\
             &= \frac{a^2 + 5}{a^2 + 5}
\end{align*}
\]
(b) Give an accurate sketch and find the domain and range of the function.

\[ f(x) = \begin{cases} 
  x + 6, & x \leq -4 \\
  2x - 4, & x > -4
\end{cases} \]
1.3 Functions

Example 8 (An Application)

The cost per unit of a certain radio model is $60. The manufacturer charges $90 per unit for orders of 100 radios or less. To encourage large orders, the manufacturer reduces the charge by $0.15 per radio for each unit ordered in excess of 100.

Ex: If 130 radios are ordered, then the charge per radio would be …

Our goal is to write Profit as a function of the number of radios ordered, x.
1.3 Functions

Example 9 (Another Application)

Write the area of a circle as a function of its circumference.

Use your function to find the area of a circle with a circumference of 27.3 inches.