1.7 Inverse Functions

Inverses – “Undo-ing” the function

\((f^{-1} \circ f)(x) = x\) for all \(x\) in the domain of \(f\)

and

\((f \circ f^{-1})(x) = x\) for all \(x\) in the domain of \(f^{-1}\)

\(f(\tilde{g}(x)) = x\)

\(\tilde{g}^{-1}(f(x)) = x\)
\[ f(x) = x + 2 \]
\[ y = x + 2 \]
\[ x = y + 2 \]
\[ x - 2 = y = f^{-1}(x) \]

inverse.
1.7 Inverse Functions

Example 1  Is \( g(x) \) the inverse function of \( f(x) \)?

\[
f(x) = 5x - 2 \quad \text{and} \quad g(x) = \frac{1}{5}(x + 2)
\]

\[
(f \circ g)'(x) = f(g'(x)) = f\left(\frac{1}{5}(x + 2)\right)
\]

\[
= 5\left(\frac{1}{5}(x + 2)\right) - 2
\]

\[
= x + 2 - 2
\]

\[
(g^{-1} \circ f)(x) = g\left(f^{-1}(x)\right) = g\left(\frac{1}{5}(5x - 2)\right)
\]

\[
= \frac{1}{5}(5x - 2 + 2)
\]

\[
= \frac{1}{5}(5x)
\]

\[
= x
\]

Yes, they are inverse.
1.7 Inverse Functions

Example 2  Is \( g(x) \) the inverse function of \( f(x) \) ?

\[
f(x) = \frac{x - 3}{2} \quad \text{and} \quad g(x) = \frac{2}{x - 3}
\]
1.7 Inverse Functions

The BIG question when does a function \( f(x) \) have an inverse that is also a function? To answer this, consider the function …

\[ f'(x) = x^2 - 2 \]
1.7 Inverse Functions

Example 3  Determine whether the following functions are one to one. For the ones that are, find the inverse function

\( f(x) = x^3 + 2 \)
1.7 Inverse Functions

Example 4  Determine whether the following functions are one to one. For the ones that are, find the inverse function

\[ f(x) = \sqrt{5x - 1} \]
1.7 Inverse Functions

**Example 5** Determine whether the following functions are one to one. For the ones that are, find the inverse function.

\[ f(x) = \sqrt{4 - x^2} \]
1.7 Inverse Functions

Example 6  Determine whether the following functions are one to one. For the ones that are, find the inverse function

\[ f(x) = (x - 3)^2 + 2 \]