Find the real solutions of the following equation:

\[ ax^2 + bx + c = 0 \]

where:

- \( a = 3 \)
- \( b = -2 \)
- \( c = 1 \)

Solving for \( x \):

1. \( \Delta = b^2 - 4ac \)
2. \( x = \frac{-b \pm \sqrt{\Delta}}{2a} \)

\[ \Delta = (-2)^2 - 4(3)(1) = 4 - 12 = -8 \]

Since \( \Delta < 0 \), there are no real solutions.

The zeros of the equation are complex numbers.

Possible roots:

- \( x = 1, 2, 4, 8 \)

Zeros:

- \( x = \frac{-5 \pm \sqrt{67}}{2} \)
- \( x = -1 \)

The roots can be found using the quadratic formula:

\[ x = \frac{-b \pm \sqrt{\Delta}}{2a} \]

\[ \Delta = 3^2 - 4(3)(1) = 9 - 12 = -3 \]

\[ x = \frac{2 \pm \sqrt{-3}}{6} \]

Simplifying:

\[ x = \frac{1 \pm \sqrt{3}i}{3} \]

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$4x^3 + 23x^2 + 34x - 10$

\[\begin{array}{cccc}
4 & 23 & 34 & -10 \\
-12+4i & 37-i & 10 \\
\end{array}\]

\[\begin{array}{cccc}
4 & 11+4i & -3-i & 0 \\
-12-4i & 3+i & 4 & -1 & 0 \\
\end{array}\]

\[(-3+i)(11+4i)\]
\[(-3+i)(37-i)\]
\[(-3+i)(-3-i)\]

\[4x = 1\]
\[x = \frac{1}{4}\]
$P_{36}^{5}$

$46$

$50$

$36$

$4, \pm 3i$

$
\beta(x) = (x-4)(x-3i)(x+3i)
$

$= (x-4)(x^2 + 9)$

$\beta(x) = x^3 - 4x^2 + 9x - 36$
\[ f(x) = x^3 + x^2 + 9x + 9 \]

\[ \begin{array}{cccc}
3i & 1 & 1 & 9 \\
-3i & 1+3i & 3i & -9+3i & -9 \\
\end{array} \]

\[ \chi + 1 = 0 \]
\[ \chi = -1 \]

\[ \pm 3i, -1 \]
\[ f(x) = x^3 + 4x^2 + 14x + 20 \]

-1 - 3i
-1 - 3i
-1 + 3i

\[ \begin{array}{cccc}
1 & 4 & 14 & 20 \\
-1 - 3i & -12 - 6i & -20 \\
-1 + 3i & 2 - 6i & 0 \\
-1 + 3i & -2 + 4i \\
\hline
2 & 0 & & \\
\end{array} \]

\[ x + 2 = 0 \]
\[ x = -2 \]
\[ f(x) = x^4 - x^2 - 20 \]

\[ \text{Root over rationals: } (x - 5)(x + 4) \]

\[ \text{Root over irrationals: } (x - \sqrt{5})(x + \sqrt{5})(x + 4) \]

\[ \text{Completely factored: } (x - \sqrt{5})(x + \sqrt{5})(x + 2i)(x - 2i) \]
\[ f(x) = x^4 + 6x^2 - 27 \]

\[ g(x) = (x^2 + 9)(x^2 - 3) \]

\[ h(x) = (x^2 + 9)(x - \sqrt{3})(x + \sqrt{3}) \]

\[ k(x) = (x + 3i)(x - 3i)(x - \sqrt{3})(x + \sqrt{3}) \]
3.5 & 3.6 RATIONAL FUNCTIONS

\[ f(x) = \frac{n(x)}{d(x)} = \frac{ax^n + \ldots}{bx^n + \ldots} \text{ in reduced form. Domain } = \{x \mid d(x) \neq 0\} \]

Example 1 \[ f(x) = \frac{1}{x} \]

- Domain \( D = \{x \mid x \neq 0\} \)
- Asymptote \( x = 0 \)
- Range \( \{y \mid y \neq 0\} \)
- Horizontal Asymptote \( y = 0 \)
- \((-\infty, 0) \cup (0, \infty)\)

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3.5 & 3.6 RATIONAL FUNCTIONS

Vertical Asymptotes: The line $x = a$ is a V.A. if $f(x) \to \infty$ or $f(x) \to -\infty$ as $x \to a$ from the left or the right.

VA’s occur where the denominator = 0

Horizontal Asymptotes: The line $y = b$ is a H.A. if $f(x) \to b$ as $x \to \infty$ or $x \to -\infty$

To find H.A.’s with $f(x) = \frac{ax^n + \ldots}{bx^d + \ldots}$

(a) If $n < d$, then $y = 0$ (the $x$-axis) is the HA.

(b) If $n = d$, then $y = \frac{a}{b}$ is the HA

(c) If $n > d$, then there is no HA
3.5 & 3.6 RATIONAL FUNCTIONS

Example 2 \( f(x) = \frac{2x^2 - 18}{x^2 - 4} \)

**Vertical Asymptote:**
\( x = 2 \), \( x = -2 \)
\( x^2 - 4 = 0 \)
\( x^2 = 4 \)
\( x = \pm 2 \)

**Horizontal Asymptote:**
\( y = 2 \)

**X-intercepts:**
\[ 2x^2 - 18 = 0 \]
\[ 2x^2 = 18 \]
\[ x^2 = 9 \]
\[ x = \pm 3 \]

\((3,0) (-3,0)\)

**Y-intercept:**
\[ \frac{18}{4} = \frac{9}{2} \]
\((0, \frac{9}{2})\)

**Decreasing:**
\((-2,-3) \cup (-3,0)\)

**Increasing:**
\((0,3) (3,\infty)\), \((-\infty,-3) \)
Example 3 (crosses HA)

\[ f(x) = \frac{-x}{x^2 - 4x + 4} \]
3.5 & 3.6 RATIONAL FUNCTIONS

Example 4 (crosses HA)

\[ f(x) = \frac{x}{x^2 + 1} \]
Example 5 (cancellation)

\[ f(x) = \frac{x + 1}{x^2 - x - 2} \]
3.5 & 3.6 RATIONAL FUNCTIONS

**Slanted Asymptotes:**

If \( m = n + 1 \) (the degree of the numerator is exactly one more than the degree of the denominator, then the function has a slanted asymptote.

The slanted asymptote can be found by performing long division.

**Example 6**  (Slanted Asymptote)

\[
f(x) = \frac{x^2 - x}{x + 1}
\]