Test #2

#8. \( f(x) = x^4 - 2x^2 \)

a.) \( y = 0 \)

b.) \( y = -1 \) (occurs \( x = -1 \) and \( x = 1 \))

c.) intervals:
   \((-1, 0)\) and \((1, \infty)\)

d.) intervals:
   \((-\infty, -1)\) and \((0, 1)\)
2.2 Solving Equations Graphically

Definitions:

The point \((a, 0)\) is called an \textbf{x-intercept} of the graph of an equation if it is a solution point of the equation. To find the \(x\)-intercept(s), set \(y\) equal to 0 and solve the equation for \(x\).

The point \((0, b)\) is called a \textbf{y-intercept} of the graph of an equation if it is a solution point of the equation. To find the \(y\)-intercept(s), set \(x\) equal to 0 and solve the equation for \(y\).

\[ \text{3 x-intercepts; 1 y-intercept} \]
Example: Find the x-intercepts and y-intercepts.

1. \( y = \frac{4}{3}x + 2 \)

\[
\begin{array}{c|c|c}
\text{x} & \text{y} & (x, y) \\
\hline
0 & 2 & (0, 2) \text{ y-intercept} \\
-\frac{3}{2} & 0 & (-\frac{3}{2}, 0) \text{ x-intercept} \\
\end{array}
\]

\[
0 = \frac{4}{3}x + 2 \\
-2 = \frac{4}{3}x \\
\left(\frac{3}{4}\right)(-2) = \frac{3}{4}\left(\frac{4}{3}x\right) \\
-\frac{3}{2} = x
\]
2. \[ y = 10 + 2(x - 2) \]

\[ y = 10 + 2x - 4 \]
\[ y = 2x + 6 \]

\[
\begin{array}{c|c|c|c}
 x & y & (x,y) & y-int \\
 0 & 6 & (0,6) & \text{y-int} \\
 -3 & 0 & (-3,0) & \text{x-int} \\
 0 & 2 & (0,2) & \\
 -6 & 2x & -3 = x \\
\end{array}
\]
3. \[ y = 3 - \frac{1}{2}|x+1| \]

- **y-intercept:** \[ y = 3 - \frac{1}{2}|0+1| = 3 - \frac{1}{2} = \frac{5}{2} \]
- **x-intercepts:** \[ x = 5, -7 \] (used calculator)

\[ y = 3 - \frac{1}{2}|x+1| \]

- For \( x+1 \geq 0 \):
  \[ y = 3 - \frac{1}{2}(x+1) = 3 - \frac{1}{2}x - \frac{1}{2} \]
  \[ y = -\frac{1}{2}x + \frac{5}{2} \]
  \[ 0 = -\frac{1}{2}x + \frac{5}{2} \] \( \frac{1}{2}x = \frac{5}{2} \) \( x = 5 \)

- For \( x+1 < 0 \):
  \[ y = 3 - \frac{1}{2}(-(x+1)) \]
  \[ y = 3 + \frac{1}{2}x + \frac{1}{2} \]
  \[ 0 = \frac{1}{2}x + \frac{7}{2} \]
  \[ -\frac{7}{2} = \frac{1}{2}x \] \( \text{add} \) by \( \frac{1}{2} \) \( x = 7 \)
Definition

A **zero** of a function \( y = f(x) \) is a number \( a \) such that \( f(a) = 0 \).

The following statements are equivalent:

1. The point \((a, 0)\) is an **x-intercept** of the graph of \( y = f(x) \).

2. The number \( a \) is a **zero** of the function \( f \).

3. The **number** \( a \) is a **solution** of the equation \( f(x) = 0 \).
Example:

4. Verify the given zeros both algebraically and graphically.

\[ f(x) = x - 3 - \frac{10}{x} \]

zeros \( x = -2, 5 \)

\[ f(-2) = -2 - 3 - \frac{10}{-2} = -2 - 3 + 5 = 0 \checkmark \]

\[ f(5) = 5 - 3 - \frac{10}{5} = 5 - 3 - 2 = 0 \checkmark \]
Graphical Approximation of Solutions of an Equation

1. Write the equation in general form $f(x) = 0$, with the nonzero terms on one side of the equation and zero on the other side.

2. Use a graphing utility to graph the function $y = f(x)$. Be sure the viewing window shows all the relevant features of the graph.

3. Use the zero feature or the zoom and trace features of the graphing utility to approximate the x-intercepts of the graph.
OR

1. Let $Y_1 = \text{left side of the equation}$ and $Y_2 = \text{right side of the equation}$.

2. Use a graphing utility to graph both $Y_1$ and $Y_2$. Be sure the viewing window shows all the relevant features of the graph.

3. Use the \textit{intersect} feature of the graphing utility to approximate the intersection of $Y_1$ and $Y_2$. 
Examples: Solve the equation algebraically. Then use a graphing utility to verify the algebraic solution, using both the zero and intersect features.

Method 1

5. \( 3.5x - 8 = 0.5x \)

\[
3x - 8 = 0
\]

\[
\begin{align*}
Y_1 &= 3x - 8 \\
Y_2 &= \\
Y_3 &= \\
Y_4 &= \\
Y_5 &= \\
Y_6 &= \\
Y_7 &=
\end{align*}
\]

Zero
\( X = 2.6666667 \quad Y = 0 \)

\[
\begin{align*}
Y_1 &= 3.5x - 8 \\
Y_2 &= 0.5x \\
Y_3 &= \\
Y_4 &= \\
Y_5 &= \\
Y_6 &= \\
Y_7 &=
\end{align*}
\]

Intersection
\( X = 2.6666667 \quad Y = 1.3333333 \)
Examples: Solve the equation algebraically. Then use a graphing utility to verify the algebraic solution, using both the zero and intersect features.

7. \[ \frac{6}{x} + \frac{8}{x+5} = 3 \] 
   \[ \text{LCM} = x(x+5) \] 
   \[ \text{mult. by } x(x+5) \]

\[ x(x+5)\left(\frac{6}{x}\right) + x(x+5)\left(\frac{8}{x+5}\right) = x(x+5)/3 \]

\[ 6x + 30 + 8x = 3x^2 + 15x \]
\[ 30 + 14x = 3x^2 + 15x \]
\[ -30 - 14x = -14x - 30 \]
\[ 0 = 3x^2 + x - 30 \]
\[ 0 = (3x + 10)(x - 3) \]
\[ 3x + 10 = 0 \]
\[ 3x = -10 \]
\[ x = -\frac{10}{3} \]
\[ x - 3 = 0 \]
\[ x = 3 \]
\[
\frac{6}{x} + \frac{8}{x+5} = 3
\]

Method 0 \[ \frac{6}{x} + \frac{8}{x+5} - 3 = 0 \]

Let \( y_1 = \)

\[
\text{Method 2} \quad \frac{6}{x} + \frac{8}{x+5} = 3
\]

Let \( y_1 = \) Let \( y_2 = \)
6. \[0.60x + 0.40(100 - x) = 50\]
8. \( \sqrt{x - 4} = 8 \)
Examples: Use a graphing utility to find any points of intersection.

\[ y = \frac{1}{3}x + 2 \]

9. \[ y = -\frac{5}{2}x - 11 \]
10. $y = 2x - x^2$

2 solutions: $(0, 0)$ and $(3, -3)$

Homework Section 2.2
2.3 Complex Numbers

Definitions:
The \textbf{imaginary unit} $i$ is defined by $i = \sqrt{-1}$ and equivalently, $i^2 = -1$.

If $a$ and $b$ are real numbers, the number $a + bi$ is a \textbf{complex number}, and it said to be written in \textbf{standard form}. If $b = 0$, the number $a + bi = a$ is a \textbf{real number}. If $b \neq 0$, the number $a + bi$ is called an \textbf{imaginary number}. A number of the form $bi$ is called a \textbf{pure imaginary number}.

(Note: The set of real numbers is a subset of the set of complex numbers.)
Two complex numbers \( a + bi \) and \( c + di \), written in standard form, are equal to each other, \( a + bi = c + di \) if and only if \( a = c \) and \( b = d \).

If \( a + bi \) and \( c + di \) are two complex numbers written in standard form, then their sum, difference, and product are given by

**Sum:** \( (a + bi) + (c + di) = (a + c) + (b + d)i \)

**Difference:** \( (a + bi) - (c + di) = (a - c) + (b - d)i \)

**Product:** \( (a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i \)

\[
\begin{align*}
ac + adi + bci + bdi^2 & \quad bd(-1) \\
= (ac - bd) + (ad + bc)i
\end{align*}
\]
Examples: Write the complex number in standard form.

1. \(3 + \sqrt{-9} = 3 + \sqrt{9(-1)} = 3 + 3i\)

2. \(-3i^2 + i = -3(-1) + i = 3 + i\)
In the following, find the sum, difference, and product of the two complex numbers.

3. $11 - 2i, -3 + 6i$

\[
\begin{align*}
\text{Sum:} & \quad (11-2i) + (-3+6i) = 8 + 4i \\
\text{Difference:} & \quad (11-2i) - (-3+6i) = 14 - 8i \\
\text{Product:} & \quad (11-2i) \cdot (-3+6i) = -21 + 72i \\
\end{align*}
\]

\[
\star (11-2i) \cdot (-3+6i) = (-12)(-1) = 12
\]

\[
\begin{align*}
&= -33 + 66i + 6i - 12i^2 \\
&= -33 + 66i + 6i + 12 \\
&= -21 + 72i
\end{align*}
\]
4. \( 7 + \sqrt{-18i} + 3 + 3\sqrt{2i} \)

\[
\begin{align*}
7 + \sqrt{-18i} & \quad 3 + 3\sqrt{2i} \\
7 + 3\sqrt{i} \sqrt{2} & = 7 + 3\sqrt{2i} \\
10 + 6\sqrt{2i} & = 10 + 6\sqrt{2}\cdot i
\end{align*}
\]

\( \theta (7 + 3\sqrt{2i}) - (3 + 3\sqrt{2i}) \)

\[ = 4 + 0i = 4 \]

\( (7 + 3\sqrt{2i})(3 + 3\sqrt{2i}) \)

\[ = 21 + 21\sqrt{2i} + 9\sqrt{2i} + 9(3i)^2 \]

\[ = 3 + 30\sqrt{2i} \]
5. \(-8i; 9 + 4i\)

**Sum:** 
\[-8i + 9 + 4i = 9 - 4i\]

**Difference:** 
\[-8i - (9 + 4i) = -9 - 12i\]

**Product:** 
\[-8i(9 + 4i) = -72i - 32i^2 = 32 - 72i\]
Simplify and write the answer in standard form.

6. \(22 + (-5 + 8i) + 10i\)

7. \((1 - 2i)^2 - (1 + 2i)^2\)

Homework
Section 2.3
(up to #35)