Section P-2 (Homework)

#17b. Simplify:

\[
\frac{\frac{4}{12}(x+y)^2}{\frac{3}{9}(x+y)} = \frac{4(x+y)^2}{3}
\]

\[
\frac{4(x^2+2xy+y^2)}{3} = \frac{4x^2+8xy+4y^2}{3}
\]

\[
\frac{4}{3}x + \frac{8}{3}xy + \frac{4}{3}y^2
\]
Factoring

Definitions:
The process of writing a polynomial as a product is called **factoring**. If a polynomial cannot be factored using integer coefficients, it is called **prime** or **irreducible** over the integers.
A polynomial is **completely factored** when each of its factors is prime.
Examples: Factor completely.

\[ 4x^3 - 6x^2 + 12x = 2x(2x^2 - 3x + 6) \]

\[ 49 - 9y^2 = 7^2 - (3y)^2 \]
\[ = (7-3y)(7+3y) \]
\[9x^2 - 12x + 4 = (3x-2)^2\]
\[(3x-2)(3x-2)\]

\[25 - (z + 5)^2 = 5^2 - (2+5)^2\]
\[= \left[\frac{5-z}{5-(2+5)}\right]\left[\frac{5+z+5}{5+(2+5)}\right]\]
\[= -2(z+10)\]

\[25 - (z+5)^2 = 25 - (z^2 + 10z + 25)\]
\[= 25 - z^2 - 10z - 25\]
\[= -2(z+10)\]
\[
a^3 - b^3 = (a - b)(a^2 + ab + b^2)
\]
\[
a^3 + b^3 = (a + b)(a^2 - ab + b^2)
\]
\[
x^3 - 27 = x^3 \quad 3 \quad 3
\]
\[
= (x - 3)(x^2 + 3x + 9)
\]

check: \[
x^3 + 3x^2 + 9x - 3x^2 - 9x - 27 \quad \checkmark
\]
\[
z^3 + 125 = z^3 \quad 5^3
\]
\[
= (z + 5)(z^2 - 5z + 25)
\]
\[ t^2 - t - 6 = (t+2)(t-3) \]

Factors of -21:

\[ 2x^2 - x - 21 = (2x - 7)(x + 3) \]
More examples: Factoring by grouping and combinations.

\[ x^3 + 5x^2 - 5x - 25 \]

\[ = x^2(x+5) - 5(x+5) \]

\[ = (x+5)(x^2-5) \]

\[ x^3+4x^2-4x-16 \]

\[ = x^2(x+4) - 4(x+4) \]

\[ = (x+4)(x^2-4) \]

\[ = (x+4)(x-2)(x+2) \]
\[
5x^3 - 10x^2 + 3x - 6
\]

\[
= 5x^2(x-2) + 3(x-2)
\]

\[
= (x-2)(5x^2 + 3)
\]

\[
\frac{5x^3 + 3x - 10x^2 - 6}{x(5x^2 + 3) - 2(5x^2 + 3)}
\]

\[
= (5x^2 + 3)(x-2)
\]
P.4 Rational Expressions

Definitions:
The set of real numbers for which an expression is defined is its domain. Two expressions are equivalent if they yield the same value for all numbers in their domain.*

The quotient of two algebraic expressions is a fractional expression. The quotient of two polynomials is a rational expression.

*domain: \( \frac{x^2 - 4}{x - 2} = \frac{(x + 2)(x - 2)}{x - 2} = x + 2 \)

- domain: \( x \neq 2 \)
- domain: all real numbers

\( (-\infty, 2) \cup (2, \infty) \)
Examples:

Find the domain:

\[ 2x^2 - 5x - 2 \] (all real numbers \((-\infty, \infty)\))

\(6x^2 - 9, \ x > 0\) (restriction or this has a restricted domain \((0, \infty)\))
Find the domain:

\[
\frac{x+1}{2x+1}
\]

\[2x+1 \neq 0 \quad 2x \neq -1 \quad x \neq -\frac{1}{2}
\]

Domain: \(x \in (-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, \infty)\)

\[
\sqrt{6-x}
\]

\[6-x \geq 0 \quad \Rightarrow \quad x \leq 6
\]

Domain: \(x \leq 6 \in (-\infty, 6]\)
Reduce:

\[
\frac{x^2 + 8x - 20}{x^2 + 11x + 10} = \frac{(x+10)(x-2)}{(x+1)(x+10)} = \frac{X-2}{X+1} \quad x \neq -10
\]

\[
\frac{x^2 - 9}{x^3 + x^2 - 9x - 9} \approx \frac{X^2 - 9}{(X+1)(X^2 - 9)} = \frac{1}{X+1} \quad x \neq \pm 3
\]
\[ \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \] (in practice, divide out any common factors first)

\[ \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} \] (use procedure for multiplication)

Perform the operations and simplify:

\[
\frac{4y-16}{5y+15} \div \frac{2y+6}{4-y}
\]

\[
= \frac{4(y-4) \cdot 2(y+3)}{5(y+3)(-1(y-4))} = \frac{8}{-5} \cdot \left(-\frac{8}{5}\right)
\]

\[
y \neq -\frac{3}{4}
\]

\[
y \neq 4
\]

\[
y = -3
\]
To add or subtract rational expressions, find the LCD; convert all terms to have the LCD; add or subtract as indicated.

\[
\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8} \quad \text{LCD} = \frac{(x-2)(x+1)(x+4)}{(x-2)(x+1)(x+4)}
\]

\[
= \frac{2}{(x-2)(x+1)} \cdot \frac{(x+4)}{(x+4)} + \frac{10}{(x-2)(x+4)} \cdot \frac{(x+1)}{(x+1)}
\]

\[
= \frac{2x+8}{(x-2)(x+1)(x+4)} + \frac{10x+10}{(x-2)(x+1)(x+4)}
\]

\[
= \frac{12x+18}{(x-2)(x+1)(x+4)} \quad \text{or} \quad \frac{6(2x+3)}{(x-2)(x+1)(x+4)}
\]
Compound fraction

Simplify:

\[
\frac{x - 4}{\frac{4}{x}} = \frac{x - 4}{\frac{x^2 - 16}{4x}} = \frac{x - 4}{\frac{x^2 - 16}{4x}} = \frac{x - 4}{\frac{x^2 - 16}{4x}}
\]

\[
= (x - 4) \div \frac{x^2 - 16}{4x} = (x - 4) \cdot \frac{4x}{x^2 - 16}
\]

\[
= \frac{(x - 4)(4x)}{(x - 4)(x + 4)} = \frac{4x}{x + 4}
\]

Stop.
OR Find LCD for "sub fractions" \( \text{LCD} = 4x \)

\[
\begin{align*}
\left( \frac{x - 4}{x} \right) & \cdot \left( \frac{4x}{x - 4} \right) = \frac{(x-4)(4x)}{x^2 - 16} \\
&= \frac{(x-4)(4x)}{(x-4)(x+4)} = \frac{4x}{x+4}
\end{align*}
\]
\[ 2x(x - 5)^{-3} - 4x^2 (x - 5)^{-4} \]

\[ = 2x \frac{(x-5)^3}{(x-5)^3} - 4x^2 \frac{(x-5)^4}{(x-5)^4} \]

\[ = 2x(x-5) - 4x^2 (x-5)^3 \]

\[ = \frac{2x(x-5)}{(x-5)^4} - 4x^2 \]

\[ = \frac{2x - 10x - 4x^2}{(x-5)^4} \]

\[ = \frac{-2x(x+5)}{(x-5)^4} \]

\[ -2x(x+5)(x-5)^{-4} \]
Factor completely:

\[24 + 5z - z^2 = (8 - z)(3 + z)\]
\[12x^2 - 48 = 12(x^2 - 4) = 12(x-2)(x+2)\]

\[5x^3 + 40 = 5(x^3 + 8) = 5(x+2)(x^2 - 2x + 4)\]
Perform the indicated operations and simplify:

\[
\frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{x^2-1}
\]

\[
= \frac{2}{x+1} \cdot \frac{(x-1)}{(x-1)} + \frac{2}{x-1} \cdot \frac{(x+1)}{(x+1)} + \frac{1}{(x+1)(x-1)}
\]

\[
= \frac{2x-2+2x+2+1}{(x+1)(x-1)} = \frac{4x+1}{(x+1)(x-1)}
\]

\[
\text{LCD} = (x+1)(x-1)
\]
\[ 5x^5 - 3x^{\frac{-3}{2}} \quad \text{LCM} = x^{\frac{3}{2}} \]

\[ = 5x^5 \cdot \frac{x^{\frac{3}{2}}}{x^{\frac{-3}{2}}} - \frac{3}{x^{\frac{3}{2}}} \]

\[ = 5x^{\frac{13}{2}} - 3 \]

\[ \left(5x^5 - 3\right) x^{-\frac{3}{2}} \]

Homework: Section P.3 and P.4