Section P-1  (Homework)

#54. \[ \frac{|x-1|}{x-1} = \begin{cases} \frac{x-1}{x-1} = 1 & \text{if } x > 1 \\ \frac{-(x-1)}{x-1} = -1 & \text{if } x < 0 \text{ and } x < 1 \end{cases} \]

#94. \[ 3x^4 + \frac{2x^3}{5} \]

Terms: \[ 3x^4, \frac{2x^3}{5} = \frac{2}{5} x^3 \]

Coefficients: \[ 3, \frac{2}{5} \]
#76. \( y \) is at most two units from \( a \)

\[ |y-a| \leq 2 \]

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Homework

P-2  1-113 (every 4th problem)

means 1, 5, 9, 13, 17, etc.
Scientific Notation (Calculator display)

examples:

\[ 580,000 = 5.8 \times 10^5 \]

\[ .0013 = 1.3 \times 10^{-3} \]
Radicals and Their Properties

Definition: Let $a$ and $b$ be real numbers with $n \geq 2$ a positive integer. If $a = b^n$, then $b$ is the $n$th root of $a$.

$n = 2$ Square root
$n = 3$ Cube root

examples:

$9 = 3^2$; 3 is a square root of 9
$9 = (-3)^2$; -3 is a square root of 9
$8 = 2^3$; 2 is the cube root of 8
$-8 = (2)^3$; -2 is the cube root of -8
$10000 = 10^4$; 10 is a 4th root of 10000
Definition: Let $a$ be a real number that has at least one $n$th root. The principal $n$th root of $a$ is the $n$th root that has the same sign as $a$. It is denoted by a radical symbol.

$$\sqrt[n]{a}$$

The positive number $n$ is the index of the radical, and the number $a$ is the radicand. If $n = 2$, omit the index.

Some perfect squares: $1, 4, 9, 16, \ldots, 81, 100, \ldots$

Some perfect cubes: $1, 8, 27, 64, \ldots, 729, 1000, \ldots$
Properties of Radicals (p. 16)

1. $\sqrt[n]{a^m} = (\sqrt[n]{a})^m$

2. $\sqrt[n]{a} \sqrt[n]{b} = \sqrt[n]{ab}$

3. $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}, b \neq 0$

4. $\sqrt[n]{\sqrt[n]{a}} = \sqrt[n]{a}$

5. $(\sqrt[n]{a})^n = a$

6. $\sqrt[n]{a^n} = |a|, \text{even}$

$n$ is

$\sqrt[n]{a^n} = a, \text{odd}$
Simplest Radical Form

1. All possible factors have been removed from the radical.
2. All fractions have radical-free denominators.
3. The index of the radical is reduced.
Examples: Evaluate or simplify:

$$\sqrt{49} = 7$$

$$\sqrt{-49} \text{ not a real number}$$

$$-\sqrt{49} = -7$$

$$\frac{4\sqrt{81}}{3} = \frac{3}{3} = 1$$
$$\sqrt[3]{\frac{16}{27}} = \frac{\sqrt[3]{8 \cdot 2}}{\sqrt[3]{27}} = \frac{\sqrt[3]{8} \cdot \sqrt[3]{2}}{\sqrt[3]{27}} = \frac{2 \sqrt[3]{2}}{3}$$

$$\sqrt{54xy^4} = \sqrt{9y^4 \cdot 6x} = 3y^2 \sqrt[3]{6x}$$

$$\sqrt{54xy^2} = \sqrt{9y^2 \cdot 6x} = \sqrt{9y^2} \sqrt[3]{6x} = 3y \sqrt[3]{6x}$$
\[
\frac{5}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{5\sqrt{10}}{10} \cdot \frac{\sqrt{10}}{2} = \frac{\sqrt{10}}{2}
\]

\[
\frac{5}{\sqrt{14} - 2} \cdot \frac{\sqrt{14} + 2}{\sqrt{14} + 2} = \frac{5(\sqrt{14} + 2)}{14 - 4} = \frac{5(\sqrt{14} + 2)}{10} = \frac{\sqrt{14} + 2}{2}
\]
$b = 2 \cdot 3$

$\sqrt[6]{x^3} = \sqrt[3]{\sqrt[6]{x^3}}$

$= \sqrt[3]{x}$

$m \sqrt{n \sqrt{x}} = \sqrt[mn]{x}$

$\sqrt[3]{3} = \sqrt[3]{x}^{3/6} = x^{1/2}$

$3 \sqrt{x + 1} + 10 \sqrt{x + 1} = 13 \sqrt{x + 1}$
\[ 7\sqrt{80x} - 2\sqrt{125x} \]

\[ = 7\sqrt{6.5x} - 2\sqrt{25.5x} \]

\[ = 7 \cdot 4\sqrt{5x} - 2 \cdot 5\sqrt{5x} \]

\[ = 28\sqrt{5x} - 10\sqrt{5x} = 18\sqrt{5x} \]
Rational Exponents

Definition: If $a$ is a real number and $n$ is a positive integer such that the principal $n$th root of $a$ exists, then $a^n = \sqrt[n]{a}$ where $1/n$ is the rational exponent of $a$. Moreover, if $m$ is a positive integer that has no common factor with $n$, then

$$a^n = (a^n)^m = (\sqrt[n]{a})^m = (a^m)^n = \sqrt[n]{a^m}.$$
Examples: Evaluate or simplify:

\[
\left(\frac{9}{4}\right)^{-\frac{1}{2}} = \frac{q^{-\frac{1}{2}}}{4^{-\frac{1}{2}}} = \frac{4^{\frac{1}{2}}}{q^{\frac{1}{2}}} = \frac{2}{3}
\]

\[
(\frac{q}{4})^{-\frac{1}{2}} = \left(\frac{q}{4}\right)^{-\frac{1}{2}} = \left(\frac{4}{q}\right)^{\frac{1}{2}} = \frac{2}{3}
\]
\[-\left(\frac{1}{125}\right)^{-\frac{4}{3}} = -\frac{1}{\left(125\right)^{-\frac{4}{3}}} = -\left(1 \cdot 125^{-\frac{4}{3}}\right)\]

\[-\left(\frac{1}{125}\right)^{\frac{4}{3}} = -\left[\left(\frac{1}{125}\right)^{-1}\right]^{\frac{4}{3}} = -\left(125\right)^{\frac{4}{3}} = -5^4 = -625\]
\[
\frac{12}{8^{\frac{5}{5}}} = 8^{\frac{12}{5} - \frac{2}{5}} = 8^{\frac{10}{5}} = 8^2 = 64
\]

\[
\frac{5^{-\frac{1}{2}} \cdot 5x^\frac{5}{2}}{(5x)^\frac{3}{2}} = \frac{5^{-\frac{1}{2}} \cdot 5 \cdot 5x^\frac{5}{2}}{5^\frac{3}{2} \cdot x^\frac{3}{2}} = \left(5^{-\frac{1}{2}} \cdot 5^{\frac{3}{2}}\right)\left(x^{\frac{5}{2} - \frac{3}{2}}\right) = 5^{-1} x^1 = \frac{x}{5}
\]
Using calculators

Examples: Use a calculator to evaluate each of the following.

\[ \sqrt[3]{45^2} = \sqrt[3]{45} \]

\[ (6.1)^{-2.9} \]

\[ (2.65 \times 10^{-4})^{\frac{1}{3}} \]

Homework Section P-2

#1, 5, 9, 13
P.3 Polynomial and Factoring

Definitions:
Let $a_0, a_1, a_2, \ldots, a_n$ be real numbers and let $n$ be a nonnegative integer. A polynomial in $x$ is an expression of the form

$$a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0, a_n \neq 0.$$ 

The polynomial is of degree $n$, $a_n$ is the leading coefficient, and $a_0$ is the constant term.
Other terms:

**Standard form**: terms are listed from highest to lowest degree

**Monomial**: one term

**Binomial**: two terms

**Trinomial**: three terms

**Zero polynomial**: 0
Examples:  (Operations with polynomials)

Perform the operations and write the result in standard form

\[-(5x^2 - 1) - (-3x^2 + 5)\]
\[= -5x^2 + 1 + 3x^2 - 5\]
\[= -2x^2 + 1 - 2(x^2 + 2)\]

\[(15.6x^4 - 18x - 19.4) - (13.9x^4 - 9.2x + 15)\]
\[= 15.6x^4 - 18x - 19.4 - 13.9x^4 + 9.2x - 15\]
\[= 1.7x^4 - 8.8x - 34.4\]
\[-4x(3 - x^3)\]

\[= -12x + 4x^4\]

\[= (7x - 2)(4x - 3)\]

\[= 28x^2 - 21x - 8x + 6\]

\[= 28x^2 - 29x + 6\]
Special Products

1. Sum and Difference of Two Terms

\[(u - v)(u + v) = u^2 - v^2\]

*Note: \(u^2 + v^2\) does not factor*

2. Square of a Binomial

\[(u + v)^2 = u^2 + 2uv + v^2\]

\[\ast(u - v)^2 = u^2 - 2uv + v^2\]

\[\ast(u-v)^2 = (u-v)(u-v) = u^2 - uv - uv + v^2\]

\[= u^2 - 2uv + v^2\]

*Note: \((u-v)^2 \neq u^2 - v^2\)*
3. Cube of a Binomial

\[(u + v)^3 = u^3 + 3u^2v + 3uv^2 + v^3\]

\[\Delta(u - v)^3 = u^3 - 3u^2v + 3uv^2 - v^3\]

\[(u-v)^3 = (u-v)^2(u-v) = (u^2 - 2uv + v^2)(u-v)\]

\[= u^3 - u^2v - 2uv^2 + 2uv^2 + uv^2 - v^3\]

\[= u^3 - 3uv^2 + 3uv^2 - v^3\]

4. Sum and Difference of Cubes

\[\star u^3 + v^3 = (u + v)(u^2 - uv + v^2)\]

\[u^3 - v^3 = (u - v)(u^2 + uv + v^2)\]

\[(u+v)(u^2-uv+v^2) = u^3 - u^2v + uv^2 - v^2 + 2uv^2 + v^3\]

\[= u^3 + v^3\]
Examples: Find the special product:

\[(u-v)^3 = u^3 - 3uv^2 + 3u^2v - v^3\]

\[(x-2)^3\]

\[u = x, \quad v = 2\]

\[x^3 - 3x^2(2) + 3(x)^2(2^2) - 2^3\]

\[= x^3 - 6x^2 + 12x - 8\]

\[(u-v)^2 = u^2 - 2uv + v^2\]

\[\left((x+1)^2 - 1\right)^2 = \left[(x+1)^2\right]^2 \cdot 2 \cdot (x+1)^2 (1) + 1^2 \quad \text{etc.} \]

\[u = (x+1), \quad v = 1\]

\[\text{OR}\]

\[\left((x+1)^2 - 1\right)^2 = [x^2 + 2x + 1 - 1]^2 = [x^2 + 2x]^2\]

\[\sqrt{(u+v)^2} = u^2 + 2uv + v^2\]

\[u = x \cdot v = 1\]

\[= x^4 + 2(x^3(2x)) + (2x)^2\]

\[= x^4 + 4x^3 + 4x^2\]
\[(x+y)(x+y+1)(x+y-1)\]
\[= (x+y)^2 - 1 = x^2 + 2xy + y^2 - 1\]

\[(x+y)(x-y)(x^2+y^2)\]
\[= (x^2-y^2)(x^2+y^2)\]
\[= x^4 - y^4\]

Homework: Section P.2
#1-55 every 4th
15, 9, 13, ...