\[
\frac{3x^3 - 17x^2 + 15x - 25}{x - 5} = 3x^2 - 2x + 5
\]
Find any asymptotes and sketch the graph.

2. \( f(x) = \frac{3x^2 + x - 5}{x^2 + 1} \)

**Vertical:** set \( x^2 + 1 = 0 \)

no real zeros

\( \Rightarrow \) no vertical asymptote

**Horizontal:** \( y = \frac{3}{1} = 3 \quad y = 3 \)

Find helpful points:

- **y-intercept:** let \( x = 0 \); \( y = -5 \)
- **x-intercepts:** set \( 3x^2 + x - 5 = 0 \)

Use QUAD: \( x = -1.468, 1.135 \)
#36. (Page 294) In a pilot project, a rural township is given recycling bins for separating and storing recyclable products. The cost $C$ (in dollars) for supplying bins to $p\%$ of the population is given by

$$C = \frac{25,000p}{100 - p}, 0 \leq p < 100.$$

(a) Find the cost of supplying bins to 15\% of the population.

$$C = \frac{25000(15)}{(100-15)} = \frac{25000(15)}{85} = \$4411.76$$

(b) Find the cost of supplying bins to 50\% of the population.

$$C = \frac{25000(50)}{(100-50)} = \frac{25000(50)}{50} = \$25000.$$
(c) Find the cost of supplying bins to 90% of the population.

\[ C = \frac{25000 \times 90}{(100 - 90)} = \frac{25000 \times 90}{10} = 225,000. \]

(d) Use a graphing utility to graph the cost function. Be sure to choose an appropriate viewing window. Explain why you chose the values that you used in your viewing window.

(e) According to this model, would it be possible to supply bins to 100% of the residents? Explain.

No, \( p \) cannot be 100 or denominator would be 0.
3.6 Graphs of Rational Functions

Guidelines for Graphing Rational Functions

Let \( f(x) = \frac{N(x)}{D(x)} \), where \( N(x) \) and \( D(x) \) are polynomials.

1. Simplify \( f \), if possible.
2. Find and plot the y-intercept (if any) by evaluating \( f(0) \).
3. Find the zeros of the numerator (if any) by solving the equation \( N(x) = 0 \). Then plot the corresponding x-intercepts.
4. Find the zeros of the denominator (if any) by solving the equation \( D(x) = 0 \). Then sketch the corresponding vertical asymptotes using dashed vertical lines.
5. Find and sketch the horizontal asymptotes (if any) of the graph using a dashed horizontal line.
6. Plot at least one point *between* and one point *beyond* each x-intercept and vertical asymptote.
7. Use smooth curves to complete the graph between and beyond the vertical asymptotes.
Examples: Sketch the graph of the rational function by hand. Use a graphing utility to verify your graph.

1. \( g(x) = \frac{x}{x^2 - 9} \)

- **y-intercept**
  \[ g(0) = 0 \]
  Point: (0,0)

- **x-intercepts**
  \[ \text{set } x = 0 \]
  Point: (0,0)

- **vert. asymptotes**
  \[ \text{set } x^2 - 9 = 0 \]
  \[ x = \pm 3 \]

- **horiz. Asymptote**
  \[ y = \pm \infty \]
  \[ y = 0 \]

Because the degree of \( N(x) \) is less than the degree of \( D(x) \).
2. \[ f(x) = \frac{x+4}{x^2+x-6} \]

- **y-intercept**: Let \( x = 0 \)
  \[ f(0) = \frac{4}{-6} = -\frac{2}{3} \]
  Point: \((0, -\frac{2}{3})\)

- **x-intercepts**: Set \( x+4 = 0 \)
  \[ x = -4 \]
  Point: \((-4, 0)\)

- **Vertical asymptotes**: Set \( x^2 + x - 6 = 0 \)
  \[ (x+3)(x-2) = 0 \]
  \[ x = -3, x = 2 \]

- **Horizontal asymptote**: \( y = 0 \)
  (degree of numerator is less than degree of denominator)
Definition: If the degree of the numerator is exactly one more than the degree of the denominator, the graph of the function has a \textit{slant} or \textit{oblique} asymptote. To find the equation of the slant asymptote, divide the denominator into the numerator.
Example: Sketch the graph of the rational function by hand. Use a graphing utility to verify your graph.

\[ f(x) = \frac{x^2 + 5x + 8}{x + 3} \]

- **y-intercept**: Let \( x = 0 \) \( f(0) = \frac{8}{3} \)
  - Point \((0, \frac{8}{3})\)

- **x-intercept**: Let \( x^2 + 5x + 8 = 0 \)
  - **QUAD (complex zeros)**

- **vertical asymptote**: Let \( x + 3 = 0 \) \( x = -3 \)
- **no horizontal asymptote** but has a **slant asymptote**:

\[
\frac{x + 2}{x + 3} \cdot \frac{x^2 + 5x + 8}{x^2 + 3x} - \frac{2x + 8}{2(x + 6)}
\]
\[ f(x) = \frac{x^2 + 5x + 8}{x+3} = (x+2) + \frac{2}{x+3} \]

\[ x = -3 \]

\[ y = x + 2 \]

(Vertical asymptote)

<table>
<thead>
<tr>
<th>x</th>
<th>y = x + 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
</tr>
</tbody>
</table>

Homework Section 3.6

Test Thursday, Nov. 17
9.1 Sequences and Series

Definitions:
An **infinite sequence** is a function whose domain is the set of positive integers. The function values $a_1, a_2, \ldots, a_n, \ldots$ are the **terms** of the sequence. If the domain of the function consists of the first $n$ positive integers only, the sequence is a **finite sequence**. $a_1 = f(1)$; $a_2 = f(2)$.

If $n$ is a positive integer, **factorial** is defined as $n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n$.
As a special case, zero factorial is defined by $0! = 1$.

$n! = n(n-1)(n-2)\cdots 3 \cdot 2 \cdot 1$
The sum of the first \( n \) terms of a sequence is represented by
\[
\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n
\]
where \( i \) is called the index of summation, \( n \) is the upper limit of summation, and 1 is the lower limit of summation. This is called a finite series.

If \( n = \infty \), the series is called an infinite series.

\[
\sum_{i} \quad \text{is the Greek letter sigma and} \quad \sum_{i=1}^{n} x_i \quad \text{is called sigma notation or summation notation.}
\]

*We will be concerned only with finite series.*
Properties of Sums

1. \( \sum_{i=1}^{n} c = cn, \ c \text{ a constant.} \)

2. \( \sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i, \ c \text{ a constant.} \)

3. \( \sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \)

4. \( \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \)
Examples: Use sigma notation to write the sum.

1. \[
\frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15}
\]

\[=
\sum_{i=1}^{15} \frac{5}{1+i}
\]
2. \[ \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{128} = \sum_{i=1}^{8} (-1)^{i-1} \frac{1}{2^{i-1}} \]

Note: or \[ \sum_{j'=0}^{7} (-1)^{j'} \frac{1}{2^{j'}} \]

3. \[ \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{10 \cdot 12} \]

\[ \sum_{i=1}^{10} \frac{1}{i(i+2)} \]
Examples: Find the sum.

4. \[ \sum_{i=1}^{6} (3i - 1) \]
   \[ = (3 \cdot 1 - 1) + (3 \cdot 2 - 1) + (3 \cdot 3 - 1) + (3 \cdot 4 - 1) + (3 \cdot 5 - 1) + (3 \cdot 6 - 1) \]
   \[ = 2 + 5 + 8 + 11 + 14 + 17 = 57 \]

5. \[ \sum_{i=1}^{5} 6 = 6(5) = 30 \]

6. \[ \sum_{k=2}^{5} (k + 1)(k - 3) \]
   \[ = (2+1)(2-3) + (3+1)(3-3) + (4+1)(4-3) + (5+1)(5-3) \]
   \[ = 3(-1) + 4(0) + 5(1) + 6(2) \]
   \[ = -3 + 0 + 5 + 12 = 14 \]