Section 2.4 (Homework)

#51. \(4x^2 + 16x + 17 = 0\)

\(a = 4, b = 16, c = 17\)

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-16 \pm \sqrt{16^2 - 4(4)(17)}}{2(4)}\]

\[= \frac{-16 \pm \sqrt{-16}}{8} = \frac{-16 \pm 4i}{8}\]

\[= -2 \pm \frac{1}{2}i\] complex

\[Y1=4X^2+16X+17\]

\[X=-1.276586 \quad Y=3.0932549\]
Solving a Polynomial Inequality

To determine the test intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order. The zeros of a polynomial are its **critical numbers**.

2. Use the critical numbers to determine the test intervals.

3. Choose one representative $x$-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every $x$-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every $x$-value in the interval.
Examples:  Solve.

9. \[6(x + 2)(x - 1) > 0\]

**Critical numbers:** -2 and 1

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Test value</th>
<th>Sign of polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>((-\infty, -2))</td>
<td>-3</td>
<td>((\cdot)(\cdot)) = +</td>
</tr>
<tr>
<td>((-2, 1))</td>
<td>0</td>
<td>((\cdot)(\cdot)) = -</td>
</tr>
<tr>
<td>((1, \infty))</td>
<td>2</td>
<td>((\cdot)(\cdot)) = +</td>
</tr>
</tbody>
</table>

\[6(x + 2)(x - 1) > 0\] when \(x < -2\) or when \(x > 1\).

\[
\begin{array}{cc}
\text{+++++} & \text{0----------0+++++++} \\
\hline
-2 & 1
\end{array}
\]

\((-\infty, -2)\) and \((1, \infty)\)
10. \(4x^2 - x^4 \leq 0\)

\[x^2(4-x^2) = x^2(2-x)(2+x) \geq 0\]

Critical numbers: 0, 2, -2

\[\begin{array}{c|ccc}
\text{intervals} & \text{represents} & \text{sign} \\
\hline
(-\infty, -2) & -3 & + & + \\
(-2, 0) & -1 & + & + \\
(0, 2) & -1 & + & - \\
(2, \infty) & 3 & + & - \\
\end{array}\]

\[2(2-x)(2+x) \leq 0\]

\[4x^2 - x^4 \leq 0 \text{ for } x \leq -2 \text{ and } x \geq 2\]

And also \(x = 0\)
Solving Rational Inequalities

Write with 0 on one side of the inequality and with a single fraction on the other side of the inequality. Find the critical numbers for both the numerator and denominator. Proceed as for polynomial inequalities.
Example: Solve.

11. \[ \frac{5+7x}{1+2x} < 4 \]

\[ \frac{5+7x}{1+2x} - 4 < 0 \]
\[ \frac{5+7x-4-8x}{1+2x} < 0 \]
\[ \frac{1-x}{1+2x} < 0 \]

Critical numbers are \( x = -1/2 \) and 1

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<td>((-\infty,-\frac{1}{2}))</td>
<td>-1</td>
<td>(\frac{1}{1} = -)</td>
</tr>
<tr>
<td>((-\frac{1}{2},1))</td>
<td>0</td>
<td>(\frac{1}{1} = +)</td>
</tr>
<tr>
<td>(1,(\infty))</td>
<td>2</td>
<td>(\frac{1}{1} = -)</td>
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</table>

The solution is \( (-\infty,-\frac{1}{2}) \) and \( (1,\infty) \).
Example: Solve.

\[ \frac{5 + 7x}{1 + 2x} \leq 4 \]

\[ \frac{5 + 7x - 4}{1 + 2x} \leq 0 \]

\[ \frac{1 - x}{1 + 2x} \leq 0 \]

1 - x = 0 \quad 1 = x

1 + 2x = 0 \quad 2x = -1 \quad x = -\frac{1}{2}

Critical numbers are x = -1/2 and 1

We know \( \frac{1 - x}{1 + 2x} < 0 \) on \((-\infty, -\frac{1}{2})\)

and \((1, \infty]\)

Where is \( \frac{1 - x}{1 + 2x} = 0 \) where x = 1

Solution \[ \frac{1}{2} \]

or \[ \frac{-1}{2} \]

Finish homework in Sec. 2.5
3.1 Quadratic Functions

Definitions

Let $n$ be a nonnegative integer and let $a_n, a_{n-1}, \ldots, a_2, a_1, a_0$ be real numbers with $a_n \neq 0$. The function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0$$

is called a polynomial function of $x$ with degree $n$. 
Let \( a, b, \) and \( c \) be real numbers with \( a \neq 0 \).

The function given by \( f(x) = a \) has degree 0 and is called a **constant function**.

The function given by \( f(x) = ax + c \) has degree 1 and is called a **linear function**.
The function given by \( f(x) = ax^2 + bx + c \) has degree 2 and is called a **quadratic function**. The graph of a quadratic function is a **parabola**.

- \( a > 0 \) (parabola opens up)
  
  \[ f(x) = x^2 \]

- \( a < 0 \) (parabola opens down)
  
  \[ f(x) = -x^2 + 2x + 3 \]
Example:

1. Sketch the following functions on the same coordinate system.

\[ f(x) = x^2 \]

\[ f(x) = 3x^2 \]

\[ f(x) = (x-3)^2 \]

\[ f(x) = (x+2)^2 + 3 \]

2 units left and 3 units up
Definitions
All parabolas are symmetric with respect to a line called the **axis of symmetry** or the **axis**. The point where the parabola intersects the axis of symmetry is called the **vertex**, which is either the minimum or the maximum value of the function.
Definition
The quadratic function given by
\[ f(x) = a(x-h)^2 + k, a \neq 0 \]
is in standard form. The graph of \( f \) is a parabola whose axis is the vertical line \( x = h \) and whose vertex is the point \( (h, k) \). If \( a > 0 \), the parabola opens upward and if \( a < 0 \), the parabola opens downward.
Examples: Sketch the graph of the quadratic equation. Identify the vertex and x-intercepts. Write each equation in standard form.

2. \( f(x) = -(x^2 + x - 30) \)

\[
f(x) - 30 = -(x^2 + x)
\]

\[
f(x) - 30 - \frac{1}{4} = -(x^2 + x + \frac{1}{4})
\]

\[
f(x) = -(x + \frac{1}{2})^2 + \frac{121}{4}
\]

standard form

vertex: \((-\frac{1}{2}, \frac{121}{4})\)  

opens down

x-intercepts are zeros  
set \( f(x) = 0 \)

\[
0 = -(x + \frac{1}{2})^2 + \frac{121}{4}
\]

\[
(x + \frac{1}{2})^2 = \frac{121}{4} \iff x + \frac{1}{2} = \pm \frac{11}{2}
\]

\[
x = -\frac{1}{2} \pm \frac{11}{2} \iff -\frac{11}{2} = -6 \quad \text{and} \quad \frac{11}{2} = 5
\]

x-intercepts: \((-6, 0), (5, 0)\)
Examples: Sketch the graph of the quadratic equation. Identify the vertex and x-intercepts. Write each equation in standard form.

2. \( f(x) = -(x^2 + x - 30) \)

\[ x\text{-coordinate of vertex is } x = -\frac{b}{2a} \]  

\[ f(x) = -x^2 - x + 30 \]

vertex: \( x = -\frac{b}{2a} = -\frac{-1}{2(-1)} = -\frac{1}{2} \)

\[ y\text{-coordinate is } f\left( -\frac{b}{2a} \right) \]

Here \( f\left( -\frac{1}{2} \right) = -\left( -\frac{1}{2} \right)^2 - \left( -\frac{1}{2} \right) + 30 = \frac{121}{4} \)

vertex \((-\frac{1}{2}, \frac{121}{4})\)

zeros: set \( f(x) = 0 \); \( -(x^2 + x - 30) = 0 \)

\[ \iff x^2 + x - 30 = 0 \]

\[ (x+6)(x-5) = 0 \quad x = -6, 5 \]

zeros \((-6, 0), (5, 0)\)
\[ a = 1; \quad b = 10; \quad c = 14 \]

3. \[ f(x) = x^2 + 10x + 14 \]

**Vertex:**

\[
\begin{align*}
    x &= \frac{-b}{2a} = \frac{-10}{2(1)} = -5 \\
    f(-5) &= (-5)^2 + 10(-5) + 14 = -11 \\
    \text{Vertex } &(-5, -11) \text{ opens up} \\
\end{align*}
\]

**Zeros:** Set \[ f(x) = 0 \]

\[
\begin{align*}
    x^2 + 10x + 14 &= 0 \\
    x &= \frac{-10 \pm \sqrt{10^2 - 4(1)(14)}}{2(1)} = \frac{-10 \pm \sqrt{44}}{2} \\
    &= -5 \pm \frac{\sqrt{11}}{2} \\
    \text{Zeros: } &(-5 + \sqrt{11}, 0) \text{ and } (-5 - \sqrt{11}, 0) \\
    &\approx (-8.317, 0) \text{ and } (-1.683, 0)
\end{align*}
\]
\[ f(x) = x^2 + 10x + 14 \]

Subtracting 14:

\[
\begin{align*}
f(x) - 14 + 25 &= x^2 + 10x + 25 \\
\frac{1}{2}(10) &= 5 \\
5^2 + 25 &= 25
\end{align*}
\]

Adding 11:

\[
\begin{align*}
f(x) + 11 &= (x+5)^2 \\
-11 &= -11
\end{align*}
\]

Therefore:

\[ f(x) = (x+5)^2 - 11 \]

Homework

Section 3.1
(by Nov. 3)
3.2 Polynomial Functions of Higher Degree

Graphs of Polynomial Functions

Continuous (all polynomials)

Discontinuous
Smooth
(all polynomials)

no sharp edges

Not Smooth

(corner)

cusp
The simplest polynomials have equations of the form \( y = x^n \):

\begin{align*}
\text{\( n \text{ even} \)} & \quad \text{\( n \text{ odd} \)} \\
Y_2 = x^4 & \quad Y_2 = x^5 \\
x = 0.7 & \quad x = 0.7 \\
Y = 0.2401 & \quad Y = 0.16807
\end{align*}