a.) Relative maximum is \( f(x) = 2 \)

b.) Relative minimum is \( f(x) = -2 \)

c.) \((-1, 1)\) \(-1 < x < 1\)

d.) \((-\infty, -1)\) and \((1, \infty)\)
(Section 2.2 continued)

Examples: Solve the equation algebraically. Then use a graphing utility to verify the algebraic solution, using both the zero and intersect features.

7. \[
\frac{6}{x} + \frac{8}{x + 5} = 3
\]

\[
(\text{mult. by } x(x + 5))
\]

\[
\frac{6x(x + 5) + 8x(x + 5)}{x(x + 5)} = 3x(x + 5)
\]

\[
6x + 30 + 8x = 3x^2 + 15x
\]

\[
14x + 30 = 3x^2 + 15x
\]

\[
-14x - 30
\]

\[
0 = 3x^2 + x - 30
\]

\[
0 = (3x + 10)(x - 3)
\]

\[
x + 10 = 0 \quad x = -\frac{10}{3}
\]

\[
x - 3 = 0 \quad x = 3
\]
7. \[ \frac{6}{x} + \frac{8}{x+5} = 3 \]

\[0] \text{ Use zero: } \frac{6}{x} + \frac{8}{x+5} - 3 = 0 \]

Repeat to verify \( x = 3 \)

\[2] \text{ Use intersect: } \]
8. \( \sqrt{x-4} = 8 \)

\[
(\sqrt{x-4})^2 = 8^2 \\
x-4 = 64 \quad (x=68) \checkmark
\]

1. Use zero to verify:

2. Use intersect to verify:
Examples: Use a graphing utility to find any points of intersection.

\[ y = \frac{1}{3} x + 2 \]

\[ y = \frac{5}{2} x - 11 \]
10. \( y = 2x - x^2 \)

Shortcut to check \((0, 0)\)

Homework 2.2
2.3 Complex Numbers

Definitions:
The imaginary unit $i$ is defined by $i = \sqrt{-1}$ and equivalently, $i^2 = -1$.

If $a$ and $b$ are real numbers, the number $a + bi$ is a complex number, and it said to be written in standard form. If $b = 0$, the number $a + bi = a$ is a real number. If $b \neq 0$, the number $a + bi$ is called an imaginary number. A number of the form $bi$ is called a pure imaginary number.

(Note: The set of real numbers is a subset of the set of complex numbers.)
Two complex numbers $a + bi$ and $c + di$, written in standard form, are equal to each other, $a + bi = c + di$ if and only if $a = c$ and $b = d$.

If $a + bi$ and $c + di$ are two complex numbers written in standard form, then their sum, difference, and product are given by

**Sum:** $$(a + bi) + (c + di) = (a + c) + (b + d)i$$

**Difference:** $$(a + bi) - (c + di) = (a - c) + (b - d)i$$

**Product:** $$(a + bi) \cdot (c + di) = (ac - bd) + (ad + bc)i$$

\[
ac + adi + bci + bdi^2 = (ac - bd) + (ad + bc)i
\]
Examples: Write the complex number in standard form.

1. $3 + \sqrt{-9} = 3 + \sqrt{9(-1)} = 3 + 3i$

2. $-3i^2 + i = -3(-1) + i = 3 + i$
In the following, find the sum, difference, and product of the two complex numbers.

3. $11 - 2i, -3 + 6i$

\[
\begin{align*}
(11 - 2i) + (-3 + 6i) &= 8 + 4i \\
(11 - 2i) - (-3 + 6i) &= 14 - 8i \\
(11 - 2i) \times (-3 + 6i) &= -21 + 72i
\end{align*}
\]
4. \[ 7 + \sqrt{-18}; 3 + 3\sqrt{2}i \]

\[ 7 + \sqrt{-18} = 7 + 3\sqrt{2}i \\
3 + 3\sqrt{2}i \]

\[ (7 + 3\sqrt{2}i)(3 + 3\sqrt{2}i) = 10 + 6\sqrt{2}i \]

To give answers in exact form:

\[ (7 + 3\sqrt{2}i)(3 + 3\sqrt{2}i) = 21 + 21\sqrt{2}i + 9\sqrt{2}i + 9(2) \cdot i^2 = 3 + 30\sqrt{2}i \]
Find sum, difference, and product:

5. \(-8i; 9 + 4i\)

\[-8i + (9 + 4i) = 9 - 4i\]
\[-8i - (9 + 4i) = -9 - 12i\]
\[-8i(9 + 4i) = -72i - 32i^2\]
\[= 32 - 72i\]
Simplify and write the answer in standard form.

6. \[ 22 + (-5 + 8i) + 10i \]
\[ = 17 + 18i \]

7. \[ (1 - 2i)^2 - (1 + 2i)^2 \]
\[ = (1 - 4i + 4i^2) - (1 + 4i + 4i^2) \]
\[ = 1 - 4i + 4i^2 - 1 - 4i - 4i^2 \]
\[ = -8i \]
Definition:
The pair of complex numbers $a + bi$ and $a - bi$ are called **complex conjugates**.

Note: $(a + bi)(a - bi) = a^2 + b^2$, a real number

We use conjugates to find the quotient of two complex numbers. 

\[
\frac{11 - 2i}{-3 + 6i} = \frac{-3 - 6i}{-3 + 6i} \cdot \frac{11 - 2i}{-3 + 6i} = \frac{-45 - 60i}{9 + 36} = \frac{-45 - 60i}{45} = -1 - \frac{4}{3}i
\]

or, using a calculator,

\[
\frac{11 - 2i}{-3 + 6i} = \frac{-1.33333333333i}{-1.33333333333i}
\]

Ans \{Frac \}

\[
-1 - \frac{4}{3}i
\]
Examples: Find the quotient.

8. \[ \frac{-8i}{9 + 4i} \cdot \frac{(9 - 4i)}{9 - 4i} = \frac{-72i + 32i^2}{9^2 + 4^2} = \frac{-32 - 72i}{97} = \frac{-32}{97} - \frac{72}{97}i \]

9. \[ \frac{(9 + 4i)}{(-8i)} \cdot \frac{8i}{8i} = \frac{72i + 32i^2}{-64i^2} = \frac{72i - 32}{64} = -\frac{1}{2} + \frac{9}{8}i \]

Or: \[ (9 + 4i) \cdot \frac{i}{(-8i)} = \frac{9i + 4i^2}{-8i^2} = \frac{9i - 4}{8} = -\frac{1}{2} + \frac{9}{8}i \]
Powers of $i$: 

\[ i^1 = i, i^2 = -1, i^3 = -i, i^4 = 1 \]

\[ i^5 = i, i^6 = -1, i^7 = -i, i^8 = 1 \quad \text{etc.} \]
Example: Write as $i$, $-1$, $-i$, or $1$:

\[
\frac{9}{4\sqrt[3]{37}} = i^{37} = (i^4)^9 \cdot i = 1 \cdot i = i
\]

10. \[
\frac{15}{4\sqrt[62]{62}} = i^{62} = (i^4)^{15} \cdot i^2 = 1(-1) = -1
\]
Skip Fractals and the Mandelbrot Set, but look at Example 6, **Plotting Complex Numbers**.

**Definition:**
The **magnitude** of a complex number \(a + bi\) is \(\sqrt{a^2 + b^2}\) and is the distance from the origin to \(a + bi\) on the complex plane.

![Complex Plane Diagram](https://via.placeholder.com/150)
Examples:

11. Plot each of the following complex numbers: $3 + 2i, -4i, -2 - 5i$ and find the magnitude of each.

- **Magnitude of $3 + 2i$**
  \[ |3 + 2i| = \sqrt{3^2 + 2^2} = \sqrt{13} \]

- **Magnitude of $-4i$**
  \[ |-4i| = \sqrt{0^2 + (-4)^2} = 4 \]

- **Magnitude of $-2 - 5i$**
  \[ |-2 - 5i| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{29} \]
2 roots of \( x^3 = 8 \)

\[ x = 2, 1, 1 \]

\[ x^3 - 8 = 0 \]