Section 11.2 (Homework)

#11. \( p = 900 \) \( r = 3.75\% \) \( t = \frac{30 \text{ days}}{360} = \frac{1}{12} \)

\[ i = \frac{pr}{t} \]

\[ i = 900(0.0375)(\frac{1}{12}) = 2.81 \]

\[ \text{Calculator:} 900(0.0375)(1/12) = 2.8125 \]
13. \( p = 587 \), \( r = 0.045\% \) per day, \( t = 2 \) months

\[ r = \frac{0.0045}{\text{day}} = \frac{60}{\text{days}} \]

\[ i = prt \]

\[ = 587 \times 0.00045 \times 60 = 15.85 \]
1. NUMBER OF DAYS BETWEEN EFFECTIVE DATE AND PARTIAL PAYMENT =
   365 - 196 = 169

2. INTEREST ON PARTIAL PAYMENT DATE = PRINCIPAL \times RATE \times (NO. OF
   DAYS IN #1) = \frac{360}{360}
   \[ i = \frac{9000 \times 0.06 \times 169}{360} = \$247.50 \]

3. PRINCIPAL PAID ON PARTIAL PAYMENT DATE = PARTIAL PAYMENT -
   INTEREST PAID = \$4372.50 - \$247.50 = \$3752.50

4. NEW PRINCIPAL = ORIGINAL PRINCIPAL - AMOUNT PAID IN #3 =
   \$5247.50
   \[ 9000 - 3752.50 = 5247.50 \]
5. **NUMBER OF DAYS BETWEEN PARTIAL PAYMENT DATE AND MATURITY DATE = 36**

   - Dec. 27 to Dec. 31: 4 days
   - Dec. 31 to Feb. 1: 32 days

   \[ \frac{32 + 4}{360} = \frac{36}{360} \]

8. **INTEREST IN MATURITY DATE = NEW PRINCIPAL \times RATE \times (NO. OF DAYS IN #5) = \$31.49**

\[ \frac{31.49}{360} = \left( \frac{5247.50}{360} \right) \cdot (0.06) \cdot \left( \frac{36}{360} \right) \]

* * *

7. **BALANCE DUE ON MATURITY DATE = NEW BALANCE + INTEREST ON MATURITY DATE = \$5278.99**

\[ 5247.50 + 31.49 = 5278.99 \]
11.3 Compound Interest

**Investment** – use of money or capital for profit.

**Fixed investment** – amount invested as principal is guaranteed and interest is computed at a fixed rate. (ex. Savings accounts, certificates of deposits, government savings bonds)

**Variable investment** – neither the principal nor the interest is guaranteed. (ex. Stocks, mutual funds, commercial bonds)
Compound interest – interest that is computed on the principal and any accumulated interest.

\[ A = p\left(1 + \frac{r}{n}\right)^{nt} \]

with \( A = \) amount at time \( t \)

\( p = \) principal
\( r = \) annual rate of interest
\( n = \) number of periods/year
\( t = \) number of years

compounded annually \( n = 1 \)
compounded quarterly \( n = 4 \)
compounded monthly \( n = 12 \)
compounded daily \( n = 360 \)
Examples:

Use $A = p\left(1 + \frac{r}{n}\right)^{nt}$ to compute:

1. The amount of the investment if $3000$ is invested for 5 years at 5% compounded annually.

\[
p = 3000; \quad r = 5\% = 0.05; \quad n = 1; \quad t = 5
\]

\[
A = p\left(1 + \frac{r}{n}\right)^{nt}
\]

\[
= 3000\left(1 + \frac{0.05}{1}\right)^{5} = 3000(1.05)^5
\]

\[
= 3828.84
\]
Suppose $3000 is invested for 5 years at 5\%:

\[ i = \frac{p \cdot r \cdot t}{100} = 3000 \cdot 0.05 \cdot 5 = 750 \]

\[ A = p + i = 3000 + 750 = 3750. \]

a) Simple interest:

b) Annual compounding: \[ A = $3828.84 \]

c) Quarterly compounding:

\[ A = 3000 \left(1 + \frac{0.05}{4}\right)^{4 \cdot 5} = 3000 \left(1 + \frac{0.05}{4}\right)^{20} \]

\[ A = $3846.11 \]
2. The amount of the investment if $5000 is invested for 10 years at 6.75% compounded daily (n=360).

\[ p = 5000 \quad r = \frac{6.75}{360} \quad t = 10 \quad n = 360 \]
\[ = 0.0675 \]

\[ A = p \left(1 + \frac{r}{n}\right)^{nt} = 5000 \left(1 + \frac{0.0675}{360}\right)^{3600} \]
\[ = \$9819.54 \]
Effective annual yield (or annual percentage yield APY) is the simple interest rate that gives the same amount of interest as a compound rate over the same period of time.

[Compute $1(1 + \frac{r}{n})^{n(1)} - 1$: note that $p = 1$ and $t = 1$.]

$$\text{APY} = \left[ 1 \left( 1 + \frac{r}{n} \right)^{n(1)} - 1 \right] \times 100\% = \_\%$$
3. The effective annual yield if money is invested at 7.5% compounded monthly. $r = 7.5\% = 0.075; n = 12$

$$\text{APY} = \left(1 + \frac{r}{n}\right)^n - 1$$

$$= \left(1 + \frac{0.075}{12}\right)^{12} - 1 = \left(1 + \frac{0.075}{12}\right)^{12} - 1$$

$$= 0.0776325989$$

Convert to percent:

$$= 0.07763\ldots \approx 7.8\%$$

Homework 11.3 syllabus up to #35
If you want to have a certain amount of money $A$ in $t$ years, the amount $p$ which would have to be invested now is called the present value.

$$p = \frac{A}{(1+\frac{r}{n})^n}$$

with $p = \text{present value}$

$A = \text{amount of money required in the future}$
4. Buddy wants to invest some money now to buy a new tractor in the future. If he wants to have $30000 available in 5 years, how much does he need to invest now in a CD paying 5.15% interest compounded monthly?