Section 7.2 (Homework)

#14: \(y + 3x - 4 = 0 \iff y = -3x + 4\)

2) \(2x - y = 7\) (2nd eqn.)

\[2x - (-3x + 4) = 7\]

\[2x + 3x - 4 = 7\]

\[5x = 11\]

\[x = \frac{11}{5}\]

\[y = -3\left(\frac{11}{5}\right) + 4\]

\[y = -\frac{33}{5} + \frac{20}{5} = -\frac{13}{5}\]

Solution: \((x, y) = \left(\frac{11}{5}, -\frac{13}{5}\right)\)

\(\approx (2.2, -2.6)\)

Check:

\[
\frac{(-13/5)+3(11/5)-4}{4}
\]

\[2(11/5) - (-13/5)\]
#11. \(y + 3x - 4 = 0\)

*2x - y = 7 \( \quad \ldots \quad x = \frac{11}{5}\)

start over

solve \& for \(x\): \(2x - y = 7\)

\(2x = y + 7\) \(x = \frac{y}{2} + \frac{7}{2}\)

Substitute into 1st eqn.

\(y + 3x - 4 = 0\)

\(y + 3\left(\frac{y}{2} + \frac{7}{2}\right) - 4 = 0\)

\(y + \frac{3}{2}y + \frac{21}{2} - 4 = 0\) \((\text{mul. by } 2)\)

\(2y + 3y + 21 - 8 = 0\)

\(5y + 13 = 0\) \(5y = -13\) \(y = -\frac{13}{5}\)
#21.\hspace{1cm} \begin{align*} \hspace{1cm} x &= 2y + 3 \\ y &= 3x - 1 \end{align*}

substitute into 2nd equation
\begin{align*} y &= 3(2y+3) - 1 \\ y &= 6y + 9 - 1 \\ y &= 6y + 8 \\ 0 &= 5y \\ y &= -\frac{8}{5} \end{align*}

\begin{align*} x &= 2y + 3 \\ y &= -\frac{8}{5} \\ x &= 2(-\frac{8}{5}) + 3 \frac{3}{5} \\ &= \frac{-16}{5} + \frac{15}{5} \\ &= -\frac{1}{5} \\ \end{align*}

\((x, y) = (\frac{-1}{5}, \frac{-8}{5})\)
7.5 Systems of Linear Inequalities

Procedure for Solving a System of Linear Inequalities (p. 418)

1. Select one of the inequalities. Replace the inequality symbol with an equal sign and draw the graph of the equation. Draw the graph with a dashed line if the inequality is \(<\) or \(>\) and with a solid line if the inequality is \(\leq\) or \(\geq\).

2. Select a test point on one side of the line and determine whether the point is a solution to the inequality. If so, shade the area on the side of the line containing the point. If the point is not a solution, shade the area on the other side of the line.

3. Repeat steps 1 and 2 for the other inequality.

4. The intersection of the two shaded areas and any solid line common to both inequalities form the solution set to the system of inequalities.
Examples: Graph the system of linear inequalities and indicate the solution set.

\[ y \leq 2x - 2 \]

1. \[ y > -x + 3 \]

Write: \( y = 2x - 2 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-2</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>

"\( \leq \)" solid line

Check \((0, 0)\) in \( y \leq 2x - 2 \)

\[ 0 \leq 2(0) - 2 \]

False; shade other side

Write: \( y = -x + 3 \)

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
</tbody>
</table>

"\( > \)" dashed line

Check \((0, 0)\) in \( y > -x + 3 \)

\[ 0 > -0 + 3 \]

False; shade other side
2. \[ 3x - y \geq -6 \]

Write: \[ x + 2y = 4 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td>(0, 2)</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>(4, 0)</td>
</tr>
</tbody>
</table>

"\geq" solid line

Check (0, 0): \[ x + 2y \geq 4 \]
0 + 2(0) \geq 4
false; shade other side

Write: \[ 3x - y = -6 \]

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>-2</td>
<td>0</td>
<td>(-2, 0)</td>
</tr>
</tbody>
</table>

"\geq" solid line

Check (0, 0): \[ 3x - y \geq -6 \]
3(0) - 0 \geq -6
0 \geq -6 true; shade this side
3. \[
\begin{align*}
&x \leq 0 \\
&y \leq 0 \\
&y \geq -3
\end{align*}
\]

Line

\[x=0 \text{ (y-axis)}\]

\[y=0 \text{ (x-axis)}\]

\[y=-3\]

\[(0,-3)\]
Try:

$$3x - y \leq 6$$

4. $$x - y > 4$$

Write: $$3x - y = 6$$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>6</td>
<td>(0, 6)</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>(2, 0)</td>
</tr>
</tbody>
</table>

"≤" solid line

Check (0,0) in $$3x - y \leq 6$$

$$3(0) - 0 \leq 6$$

$$0 \leq 6$$ true;

shade this side

Write: $$x - y = 4$$

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>(4, 0)</td>
</tr>
</tbody>
</table>

">" dashed line

Check (0,0) in $$x - y > 4$$

$$0 - 0 > 4$$ false;

shade other side

Homework Section 7.5
7.6 Linear Programming

A linear programming problem usually has many variables and is so lengthly it must be solved on a computer using the simplex algorithm.

**Constraints** – restrictions represented as linear inequalities.

**Feasible region** – region bounded on all sides which is the graph of the system of linear inequalities.

**Vertices** – points where two or more boundaries intersect.
In a linear programming problem, we want to maximize or minimize some quantity, say $K$. We will express $K$ as $K = Ax + By$, the objective function.

Fundamental Principle of Linear Programming – If the objective function $K = Ax + By$ is evaluated at each point in a feasible region, the maximum and minimum values of the equation occur at vertices of the region.