7.1 Systems of Linear Equations

A solution to a system of linear equations (also called simultaneous linear equations) is the ordered pair or pairs that satisfy all equations in the system.

A system of equations is consistent if there is any solution.

A system of equations is inconsistent if there is no solution.

A system of equations is dependent if there is an infinite number of solutions.
unique solution

parallel lines
no solution

consistent

inconsistent

infinite number of solutions
(every point on the line)

consistent but dependent
Examples: Approximate the solution to the given systems of equations by graphing. If the system does not have a single ordered pair as a solution, state whether the system is inconsistent or dependent.

1. \[3x - y = 3\]
   \[3y - 4x = 6\]

\[
\begin{array}{c|c|c}
 x & y & (x, y) \\
\hline
 0 & -3 & (0, -3) \\
 1 & 0 & (1, 0) \\
\end{array}
\]

\[
\begin{array}{c|c|c}
 x & y & (x, y) \\
\hline
 0 & 2 & (0, 2) \\
 -\frac{3}{2} & 0 & (-\frac{3}{2}, 0) \\
\end{array}
\]

\[
\begin{align*}
3x - y &= 3 \\
y &= -3x + 3 \\
3y - 4x &= 6 \\
y &= \frac{4}{3}x + 2
\end{align*}
\]
L₁ \( y = -\frac{1}{3}x - 4 \) \( \text{slope} = -\frac{1}{3} \)

L₂ \( 3y - x = 4 \) \( \text{solve for } y \)
\[ 3y = x + 4 \]
\[ y = \frac{1}{3}x + \frac{4}{3} \] \( \text{slope} = \frac{1}{3} \)

Two lines with the same slope are parallel \( \Rightarrow \) no intersection (no solution)

system is inconsistent
7.2 Solving Systems of Equations by the Substitution and Addition Methods

Substitution Method (See “Procedure for Solving a System of Equations Using the Substitution Method” on p. 391)
Examples: Solve the system of equations by the substitution method. If the system does not have a single ordered pair as a solution, state whether the system is inconsistent or dependent.

\[ y = 5x + 7 \]  
(substitute into 2nd equation)

1. \[ y = 2x + 1 \]

\[
\begin{align*}
5x + 7 &= 2x + 1 \\
-2x - 7 &= -2x - 7 \\
3x &= -6 \\
x &= -2
\end{align*}
\]

Solution: \((x, y) = (-2, -3)\)

Check: \[ y = 5x + 7 \]
\[ y = 2x + 1 \]

\[ (-2, -3) \]

- \[ 5(-2) + 7 = -3 \]
- \[ 2(-2) + 1 = -3 \]
\[ x + y = 3 \]

2. \[ y + x = 5 \]

\[ (-x+3) + x = 5 \]

\[ -x + 3 + x = 5 \]

\[ 3 = 5 \text{ false } \Rightarrow \]

system is inconsistent

\[ y = -x + 3 \text{ (Substitute into 2nd equation)} \]
3. \[ x + 2y = 6 \]
\[
\begin{align*}
y &= 2x + 3 \\
\ast y &= 2x + 3 \\
x + 2(2x + 3) &= 6 \\
x + 4x + 6 &= 6 \\
5x &= 0 \\
x &= 0 \\
\circled{x=0} \\
\ast y &= 2(0) + 3 = 3 \\
y &= 2(0) + 3 = 3 \\
\text{Solution: } (x, y) &= (0, 3)
\end{align*}
\]

Check:
\[
\begin{align*}
x + 2y &= 6 \\
0 + 2(3) &= 6 \\
\checkmark \\
(x, y) &= (0, 3) \\
y &= 2x + 3 \\
3 &= 2(0) + 3 \\
\checkmark
\end{align*}
\]
Addition Method (See “Procedure for Solving a System of Equations by the Addition Method” on p. 394)

Examples: Solve the system of equations by the addition method. If the system does not have a single ordered pair as a solution, state whether the system is inconsistent or dependent.

\[ \begin{align*}
2x - 6y &= 8 \\
-2x + 4y &= -10
\end{align*} \]

\[ -2y = -2 \quad y = 1 \]

\[ \begin{align*}
*2x - 6y &= 8 \\
2x - 6(1) &= 8 \\
2x &= 14 \quad x = 7
\end{align*} \]

Solution: \((x, y) = (7, 1)\)

Check: \(2x - 6y = 8\)
\[-2x + 4y = -10 \quad -2(7) + 4(1) = -10 \quad \checkmark \]
5. \[4x + 3y = -1\] 
\[2x - y = -13\] (mult. by 2)

\[
\begin{align*}
4x + 3y &= -1 \\
-4x + 2y &= 26 \\
\underline{0} &
\end{align*}
\]
\[5y = 25\]  \(y = 5\)

Alternate way to solve for \(x\): (start over) eliminate \(y\) and solve for \(x\)

\[
\begin{align*}
4x + 3y &= -1 \\
6x - 3y &= -39 \\
\underline{10x} &= -40 \\
\end{align*}
\]
\[x = -4\]
\[4x + 3y = -1\]
\[2x - y = -13\]

**Solution:** \((x, y) = (-4, 5)\)

**Check:**
\[4(-4) + 3(5) = 4 \times (-4) + 3 \times 5 = -16 + 15 = -1 \quad \checkmark\]
\[2(-4) - 5 = 2 \times (-4) - 5 = -8 - 5 = -13 \quad \checkmark\]
6. \[4x - 2y = 6\]
\[4y = 8x - 12\]

\[
\begin{align*}
4x - 2y &= 6 \quad \text{(multiplied by 2)} \\
-8x + 4y &= -12
\end{align*}
\]

\[
\begin{align*}
8x - 4y &= 12 \\
-8x + 4y &= -12
\end{align*}
\]

\[0 = 0\] true dependent

one line - every point on the line is a solution

Sections 7.1, 7.2