Section 6.1 (Homework)

#21. \( \frac{1}{2} x^2 - 5x + 2 \) \( x = \frac{2}{3} \)

\[
\begin{align*}
&= \frac{1}{2} \left( \frac{2}{3} \right)^2 - 5 \left( \frac{2}{3} \right) + 2 \\
&= \frac{1}{2} \left( \frac{4}{9} \right) - 5 \left( \frac{2}{3} \right) + 2 \\
&= \frac{2}{9} - \frac{10}{3} + 2 = \frac{2}{9} - \frac{10}{3} + \frac{2}{1} \cdot \frac{9}{9} \\
&= \frac{2}{9} - \frac{30}{9} + \frac{18}{9} = -\frac{10}{9}
\end{align*}
\]
Section 6.2 (continued)

A linear (or first degree) equation in one variable is one in which the exponent on the variable is 1.

**example:** $3x - 2 = x + 4$

Equivalent equations are equations that have the same solution.
To solve an equation, isolate the variable by using the four properties of equality.

1. If \( a = b \), then \( a + c = b + c \).

2. If \( a = b \), then \( a - c = b - c \).

3. If \( a = b \), then \( ac = bc \), \( c \) is not 0.

4. If \( a = b \), then \( a/ c = b/ c \), \( c \) is not 0.
Examples: Solve the equation.

1. \[
2y - 7 = -17 \\
\frac{+7}{+7} \quad \frac{+7}{+7} \\
2y = -10 \\
\frac{2y}{2} = \frac{-10}{2} \\
y = -5 \\
\text{Check:} \quad 2(-5) - 7 = -17 \\
-10 - 7 = -17
\]

2. \[
\frac{x - 1}{5} = \frac{x + 5}{15} \\
5 \left( \frac{x - 1}{5} \right) = 15 \left( \frac{x + 5}{15} \right) \\
3x - 3 = x + 5 \\
-x + 3 = -x + 3 \\
2x = 8 \\
\frac{2x}{2} = \frac{8}{2} \quad (x = 4)
\]

Check: \[
\frac{4 - 1}{5} = \frac{4 + 5}{15} \\
\frac{3}{5} = \frac{19}{15}
\]
3. \[ \frac{x}{4} + 2x = \frac{1}{3} \]

\[ \frac{3}{12} \left( \frac{x}{4} \right) + \frac{12}{3} \cdot 2x = \frac{12}{3} \left( \frac{1}{3} \right) \]

\[ 3x + 24x = 4 \]

\[ 27x = 4 \]

\[ \frac{27x}{27} = \frac{4}{27} \]

\[ x = \frac{4}{27} \]

4. \[ 3(x + 2) + 2(x - 1) = 5x - 7 \]

\[ 3x + 6 + 2x - 2 = 5x - 7 \]

\[ 5x + 4 = 5x - 7 \]

\[ -5x = -11 \]

\[ 4 = -7 \] (false)

\[ \Rightarrow \text{no solution} \]

(\text{the equation is called a contradiction})
4. \[ 3(x + 2) + 2(x - 1) = \frac{5x + 4}{5x - 7} \]

\[ 3x + 6 + 2x - 2 = 5x + 4 \]
\[ 5x + 4 = 5x + 4 \]
\[ -5x + 4 - 5x - 4 \]
\[ 0 = 0 \]

solutions: all real numbers
Special cases:
If an equation has no solution, it is called a **contradiction**. For example, $x + 3 = x + 2$.

If every real number is a solution to an equation, it is called an **identity**. For example, $2(x + 3) = 2x + 6$. 
A proportion is a statement of equality between two ratios.

\[
\frac{a}{b} = \frac{c}{d}
\]

Example: A gallon of paint covers 825 ft\(^2\). How much paint is needed to cover a house with 5775 ft\(^2\) of surface area? \(x=\text{no. of gallons of paint required}\)

Set us the proportion as \(\frac{\text{gallons}}{\text{ft}^2}\).

\[
\frac{1}{825} = \frac{x}{5775}
\]

"cross multiply"

\[
1 \times 5775 = 825x
\]

5775 = 825 \(x\)

\[\frac{5775}{825} = x\]

7 glasses = \(x\)
6.3 Formulas

A **formula** is an equation that usually has a real-life application. (A formula generally contains more than one variable.)

A formula or a group of equations may represent a mathematical model for a real situation.

**Examples:** \( A = \pi r^2 \); \( d = rt \); \( P = 2L + 2W \)

**Subscripts:** For the variables \( x_1, x_2, x_3 \), the subscripts are 1, 2, and 3, and \( x_1 \) is not necessarily equal to \( x_2 \) etc.

Read \( x_1 \) as "x sub 1" and \( x_2 \) as "x sub 2"

**Example:** In a class, you will have 4 tests; your final grade is the simple average of the 4 tests.

\[
\text{average} = \frac{T_1 + T_2 + T_3 + T_4}{4}
\]
Examples:  Use the formula to find the value of the indicated variable for the given values.

1. \( F = ma \); find \( m \) when \( F = 40 \) and \( a = 5 \)

\[
\frac{40}{5} = m(5) \quad \Rightarrow \quad 8 = m
\]

2. \( B = \frac{703w}{h^2} \); find \( B \) when \( w = 130 \) and \( h = 67 \). (Body mass index.)

\[
B = \frac{703(130)}{67^2} = 20.35
\]
Solve for $y$.

3. \[8x - 6y = 21\]
   \[-8x\]
   \[\underline{-8x}\]
   \[-6y = -8x + 21\]
   \[-\frac{6y}{6} = -\frac{8x}{6} + \frac{21}{6}\]
   \[-y = \frac{4}{3}x - \frac{7}{2}\]

4. \[-2x + 4y = 9\]
   \[+2x\]
   \[\underline{+2x}\]
   \[4y = 2x + 9\]
   \[\frac{4y}{4} = \frac{2x}{4} + \frac{9}{4}\]
   \[y = \frac{1}{2}x + \frac{9}{4}\]
Exponential equations have the form: \( y = a^x \), with \( a > 0 \) and \( a \neq 1 \).

Exponential growth or decay is represented by the equation \( P = P_0a^{kt} \), where

\[
\begin{align*}
P_0 &= \text{initial amount present (when } t = 0) \\
P &= \text{amount present at time } t \\
a \text{ and } k &= \text{numbers dependent on the particular problem}
\end{align*}
\]

(If \( k > 0 \), there is exponential growth; if \( k < 0 \), there is exponential decay.)
Example: If 20 mg of carbon 14 (C\textsubscript{14}) are originally present in an animal bone, how much will remain at the end of 500 years if \( P = P_0 \times 2^{-\frac{t}{5600}} \).

\[
P_0 = 20 \text{ mg}, \quad t = 500
\]

\[
P = 20 \times 2^{-\frac{500}{5600}} = 18.8 \text{ mg}
\]

\[
\approx 18.8 \text{ mg}
\]
1st bottle $157,000 London 1985
2nd collector investigating

Homework: Section 6.2 & 6.3