1. (5% each) Solve the equation and check your answer either algebraically or graphically.

   a. \[3(5x - 4) + 5(x - 6) = 0\]

\[
15x - 12 + 5x - 30 = 0 \\
20x - 42 = 0 \\
20x = 42 \implies x = \frac{42}{20} = \frac{21}{10}
\]

   b. \[5 = -1 + \frac{3}{x} \quad \text{(mult. by } x)\]

\[
5x = -1 \cdot x + \frac{3}{x} \cdot x \\
5x = -x + 3 \\
+ x + x \\
6x = 3 \\
\frac{6x}{6} = \frac{3}{6} \implies x = \frac{1}{2}
\]
2.\ (5\%\ each)\ Solve\ by\ the\ Quadratic\ Formula.\ Write\ your\ answer\ in\ both\ exact\ form\ and\ using\ a\ decimal\ approximation.

\[ x^2 + 6x + 4 = 0 \]

\[ a = 1; \quad b = 6; \quad c = 4 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \]

exact \[ \frac{-3 \pm \sqrt{5}}{2} \]

approximate \[ -1.764, -5.236 \]

\[ x = -\frac{6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)} = -\frac{6 \pm \sqrt{20}}{2} \]

\[ = -\frac{6 \pm \sqrt{4 \cdot 5}}{2} = -\frac{6 \pm 2\sqrt{5}}{2} \]

\[ = -\frac{6 + 2\sqrt{5}}{2}, \quad -\frac{6 - 2\sqrt{5}}{2} \]

or \[ -1.764, -5.236 \]
3. (5% each) Perform the operation and write the result in the standard form \( a + bi \). 

\[
\begin{align*}
\text{a.} & \quad \frac{(5-2i)(2-3i)}{(2+3i)(2-3i)} & \quad \frac{4-19i}{13} \\
& = \frac{10-15i-4i-6i^2}{2^2+3^2} & = \frac{10-15i-4i+6i^2}{2^2+3^2} \\
& = \frac{10-19i+6(-1)}{13} & = \frac{4-19i}{13}
\end{align*}
\]

\[
\begin{align*}
\text{b.} & \quad (3+\sqrt{16})(2-\sqrt{9}) & \quad 18-i \\
& = (3+4i)(2-3i) & \\
& = 6-9i+8i-12i^2 & = 6-9i+8i+12 & = 18-i
\end{align*}
\]
4. (5% each) Find all solutions, real and complex, algebraically. Use your calculator as an aid.

a. \( x^2 + 5x - 24 = 0 \)
   \[ (x+8)(x-3) = 0 \]
   \[ \Rightarrow x + 8 = 0 \quad x = -8 \]
   \[ x - 3 = 0 \quad x = 3 \]

b. \( 3x^3 - 75x = 0 \)
   \[ 3x(x^2 - 25) = 0 \]
   \[ 3x(x-5)(x+5) = 0 \]
   \[ \Rightarrow 3x = 0 \quad x = 0 \]
   \[ x - 5 = 0 \quad x = 5 \]
   \[ x + 5 = 0 \quad x = -5 \]

To verify solution
c. \[ \sqrt{4-x} + 2 = 0 \]

\[ \frac{-2}{\sqrt{4-x}} = -2 \]

Stop

or \[ (\sqrt{4-x})^2 = (-2)^2 \]

\[ 4-x = 4 \]

\[ -x = 0 \]

\[ x = 0 \]

Substitute \( \sqrt{4-x} = 0 \)

d. \[ 2x^2(x-1) + 6x(x-1)^{\frac{1}{3}} = 0 \]

\[ 2x(x-1)^{\frac{1}{3}}[x + 3(x-1)] = 0 \]

\[ 2x(x-1)^{\frac{1}{3}}(4x-3) = 0 \]

\[ \Rightarrow 2x = 0 \quad x = 0 \]

\[ (x-1)^{\frac{1}{3}} = 0 = [(x-1)^{\frac{1}{3}}]^3 = 0^3 \Rightarrow x-1 = 0 \Rightarrow x = 1 \]

\[ 4x-3 = 0 = x = \frac{3}{4} \]
5. (5% each) Find all real solutions, algebraically or graphically. You must show how you found your answers.

a. \(|x^2 + 3x| = x + 3\)

\[ \begin{align*}
|X^2+3X| &= X^2+3X = X+3 \\
\end{align*} \]

\[ \begin{align*}
X^2+2X-3 &= 0 \\
(X+3)(X-1) &= 0 \\
X &= -3, 1
\end{align*} \]

\[ \begin{align*}
|X^2+3X| &= -(X^2+3X) = X+3 \\
-X^2-3X &= X+3 \\
+X^2+3X &= X^2+3X
\end{align*} \]

\[ \begin{align*}
0 &= X^2+4X+3 \\
0 &= (X+1)(X+3) \\
=> X &= -1, -3
\end{align*} \]

To verify
b. \( \sqrt{x+9} = \sqrt{9-x} \)

\[
\left( \sqrt{x+9} \right)^2 = \left( \sqrt{9-x} \right)^2
\]

\[
x+9 = 9-x
\]

\[
x + x - 9 = -9 + x
\]

\[
2x = 0
\]

\[x = 0\]
You wish to use a graphing utility to find the solutions to $4x^{2/3} + 1.6 = 8x^{1/3}$ or $4x^{2/3} + 1.6 - 8x^{1/3} = 0$

a. (2% each) One way to find the solutions is to let $Y_1 = \frac{2}{3} \cdot \frac{4x + 1.6}{x^{1/3}}$ and then use the ** ZERO** command from the menu.

b. (2% each) A second way to find the solutions is to let $Y_1 = \frac{4x + 1.6}{x^{1/3}}$ and then use the **intersect** command from the menu.

c. (5%) Use either method to find the solutions. $x = 0.011$ and $5.589$.

Title: Oct 25 - 11:16 AM (8 of 17)
7. (5% each) Solve algebraically or graphically and sketch the solution on the real number line.

a. \(-4 < 3x + 2 \leq 7\)

\[
\begin{align*}
-2 &< 3x + 2 \\
-2 - 2 &< 3x \\
-6 &< 3x \\
\frac{-6}{3} &< x \\
-2 &< x \\
\frac{5}{3} &< x
\end{align*}
\]

\(\Rightarrow -2 < x \leq \frac{5}{3}\)

b. \(|x - 5| > 8\)

\[
\begin{align*}
\Rightarrow x - 5 &> 8 \\
x &> 13
\end{align*}
\] or 

\[
\begin{align*}
\Rightarrow x - 5 &< -8 \\
x &< -3
\end{align*}
\] or 

\((-\infty, -3) \cup (13, \infty)\)
c. \[ |2x+3| \leq 5 \]

\[-5 \leq 2x + 3 \leq 5\]

\[-3 \quad -3 \quad -3\]

\[-8 \leq 2x \leq 2\]

\[-\frac{8}{2} \leq \frac{2x}{2} \leq \frac{2}{2}\]

\[-4 \leq x \leq 1 \text{ or } [-4, 1]\]
\[ \frac{x+12}{x+2} - 3 \geq 0 \]

\[ \frac{x+12}{x+2} - \frac{3(x+2)}{x+2} \geq 0 \]

\[ \frac{-2x + 6}{x+2} \geq 0 \text{ or } \frac{-2(x-3)}{x+2} \geq 0 \]

Critical points: \( x = 3 \), \( x = -2 \)

<table>
<thead>
<tr>
<th>Interval</th>
<th>Test Value</th>
<th>Sign of ( \frac{-2(x-3)}{x+2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( (-\infty, -2) )</td>
<td>(-3)</td>
<td>(-)</td>
</tr>
<tr>
<td>( (-2, 3) )</td>
<td>(0)</td>
<td>()</td>
</tr>
<tr>
<td>( (3, \infty) )</td>
<td>(4)</td>
<td>(-)</td>
</tr>
</tbody>
</table>

At least \( -2, 3 \): check endpoints

\( 3 \) is ok

Solution: \( (-2, 3] \) and \( -2 < x \leq 3 \)
3. (15%) The table gives the advertising expenditures $x$ and the sales volume $y$ for a company for 6 randomly selected months. Both are measured in thousands of dollars.

<table>
<thead>
<tr>
<th>Month</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1.6</td>
<td>1.8</td>
<td>2.1</td>
<td>2.7</td>
<td>2.3</td>
<td>1.7</td>
</tr>
<tr>
<td>$y$</td>
<td>800</td>
<td>840</td>
<td>920</td>
<td>960</td>
<td>920</td>
<td>860</td>
</tr>
</tbody>
</table>

(a) Use your calculator to find a linear model for the data.

\[
y = 134.4x + 610.2
\]

(b) Sketch the scatterplot and the regression line on the same coordinate system. Label clearly.

(c) Use the model to predict sales for advertising expenditures of $2,000.

Let $x = 2.0$, then $y = 134.4(2) + 610.2$.

$y = 879.8$
3.1 Quadratic Functions

Definitions

Let \( n \) be a nonnegative integer and let \( a_n, a_{n-1}, \ldots, a_2, a_1, a_0 \) be real numbers with \( a_n \neq 0 \). The function given by

\[
f(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_1 x + a_0
\]

is called a polynomial function of \( x \) with degree \( n \).

Let \( a, b, \) and \( c \) be real numbers with \( a \neq 0 \).

The function given by \( f(x) = a \) has degree 0 and is called a constant function.

The function given by \( f(x) = ax + c \) has degree 1 and is called a linear function.

The function given by \( f(x) = ax^2 + bx + c \) has degree 2 and is called a quadratic function. The graph of a quadratic function is a parabola.
\( a > 0 \)

\[ f(x) = x^2 \]

\( a < 0 \)

\[ f(x) = -x^2 + 2x + 3 \]
Example:

1. Sketch the following functions on the same coordinate system.
   \[ f(x) = x^2 \]
   \[ f(x) = 3x^2 \]
   \[ f(x) = (x-3)^2 \]
   \[ f(x) = (x+2)^2 + 3 \]
Definitions
All parabolas are symmetric with respect to a line called the axis of symmetry or the axis. The point where the parabola intersects the axis of symmetry is called the vertex, which is either the minimum or the maximum value of the function.
Definition
The quadratic function given by
\[ f(x) = a(x - h)^2 + k, a \neq 0 \]
is in **standard form**. The graph of \( f \) is a parabola whose axis is the vertical line \( x = h \) and whose vertex is the point \((h, k)\). If \( a > 0 \), the parabola opens upward and if \( a < 0 \), the parabola opens downward.