Section 1.2 (Homework)

#3.

a) $m = 2$

b) $m = 1$ (horizontal)

c) $m = -3$

d) $-3 = \frac{3}{1}$ or $\frac{3}{-1}$
Examples: Graph each of the following on your calculator. Determine the domain and range of each function. For the last two functions, graph in both connected and dot mode.

1. \[ y = \sqrt{x + 2} \]

2. \[ f(x) = \sqrt{x + 2} \]
   \[ f(3) = \sqrt{3 + 2} = \sqrt{5} \]

 alternate notations

domain: \( x \geq -2 \)
range: \( y \geq 0 \)
\[ f(x) = \frac{x+1}{x+2} \]

domain: \( x + 2 \neq 0 \)
\[ \Rightarrow x \neq -2 \]

Connected Mode
this line is not part of graph

Dot Mode
line is removed
disadvantage: fewer points

domain: \( x \neq -2 \)

range: \( y \neq 1 \)
can't prove this yet
\[ f(x) = \frac{x^2 - 1}{x - 1} \]
\[ = \frac{(x-1)(x+1)}{x-1} = x+1, \quad x \neq 1 \]

Domain: \( x \neq 1 \)

Standard Screen

Decimal Screen

CALC 1: Value

function is not defined at \( x = 1 \)

Better graph
Definitions: If \( f \) is defined by an algebraic expression and the domain is not specified, the **implied domain** consists of all real numbers for which the expression is defined.

A **piecewise-defined function** is a function that is defined by two or more equations over a specified domain.
Example: A simple example of a piecewise-defined function is the absolute value function. We may write

\[ f(x) = |x| = \begin{cases} 
  x, & x \geq 0 \\
  -x, & x < 0 
\end{cases} \]
Example:

Given: 
\[ f(x) = \begin{cases} 
  x^2 + 2, & x \leq 1 \\
  2x^2 + 2, & x > 1 
\end{cases} \]

find \( f(-2), f(0), f(1), f(2) \).

\[
\begin{array}{c|c}
 x & f(x) = x^2 + 2 \\
\hline
-2 & 6 \\
0 & 2 \\
1 & 3 \\
2 & \hline
\end{array}
\]

\[
\begin{array}{c|c}
 x & f(x) = 2x^2 + 2 \\
\hline
-2 & 10 \\
2 & \hline
\end{array}
\]

Standard Screen Connected Mode
Example 8 (p. 105)  The Path of a Baseball

A baseball is hit at a point 3 feet above the ground at a velocity of 100 feet per second and an angle of 45 degrees. The path of the baseball is given by the function \( f(x) = -0.0032x^2 + x + 3 \) where \( y \) and \( x \) are measured in feet. Will the baseball clear a 10 foot fence located 300 feet from home plate?

Algebraic solution:

\[ f(300) = -0.0032(300)^2 + 300 + 3 = 15 \text{ ft.} \]  \( \checkmark \) clears \( 10 \text{ ft.} \)

Graphical solution:  Set an appropriate Window and graph both \( f(x) \) and the line \( y = 10 \).
Definition: The ratio \( \frac{f(x+h) - f(x)}{h} \), \( h \neq 0 \) is called the difference quotient.

Example: Find the difference quotient for the function \( f(x) = 5x - x^2 \) for \( x = 5 \).

\[
\begin{align*}
  f(x+h) &= f(5+h) = 5(5+h) - (5+h)^2 \\
  &= 25 + 5h - (25 + 10h + h^2) \\
  &= 25 + 5h - 25 - 10h - h^2 \\
  &= -5h - h^2 \\
  f(x+h) - f(x) &= f(5+h) - f(5) \\
  &= -5h - h^2 - 0 = -5h - h^2 \\
  \frac{f(x+h) - f(x)}{h} &= \frac{-5h - h^2}{h} = \frac{h(-5-h)}{h} \\
  &= -5-h
\end{align*}
\]
1.4 Graphs of Functions

Definitions: The graph of a function $f$ is the collection of ordered pairs $(x, y)$ such that $x$ is in the domain of $f$.

$x =$ the directed distance from the $y$-axis

$y =$ the directed distance from the $x$-axis

Example: Find the domain and range

Estimates: domain: $1 \leq x \leq 4$
range: $-1 \leq y \leq 3$
Vertical Line Test for Functions: A set of points in a coordinate plane is the graph of $y$ as a function of $x$ if and only if no vertical line intersects the graph at more than one point.

Examples:

- Function
- Not a function
Definitions: A function $f$ is **increasing** on an interval if, for any $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

A function $f$ is **decreasing** on an interval if, for any $x_1$ and $x_2$ in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

A function $f$ is **constant** on an interval if, for any $x_1$ and $x_2$ in the interval, $f(x_1) = f(x_2)$. 

![Diagram showing increasing, constant, and decreasing functions](image.png)
Definitions:  A function value $f(a)$ is called a **relative minimum** of $f$ if there exists an interval $(x_1, x_2)$ that contains $a$ such that $x_1 < x < x_2$ implies $f(a) \leq f(x)$.

A function value $f(a)$ is called a **relative maximum** of $f$ if there exists an interval $(x_1, x_2)$ that contains $a$ such that $x_1 < x < x_2$ implies $f(a) \geq f(x)$.

Relative minima and relative maxima are called **relative extrema**.
Example: Use a graphing utility to find any relative minimum or relative maximum values of the function and those intervals where $f$ is increasing and those intervals where $f$ is decreasing.

1. $f(x) = (x-1)^2(x+2)$

Intervals of increase and decrease ($x$'s)
- Increases on $(-\infty, -1)$ and $(1, \infty)$
- Decreases on $(-1, 1)$

Relative maximum $y = f(x) = 4$ when $x = -1$
Relative minimum $y = f(x) = 0$ when $x = 1$
3. \[ h(x) = x\sqrt{4-x} \]

\[ y = 4 - x \geq 0 \]

\[ y = -x \geq -4 \iff x \leq 4 \]

relative maximum

\[ y = f(x) \approx 3.079 \]

at \( x \approx 2.667 \)

Increasing \((-\infty, 2.667)\)

Decreasing \((2.667, 4)\)
Definition: The greatest integer function is denoted by $\|x\|$ and is defined by $f(x) = \|x\| = \text{the greatest integer less than or equal to } x$. 
Example: (p.123 #82)

The cost of sending an overnight package from New York to Atlanta is $9.80 for a package weighing up to but not including 1 pound and $2.50 for each additional pound or portion of a pound. Use the greatest integer function to create a model for the cost $C$ of overnight delivery of a package weighing $x$ pounds, where $x > 0$. Sketch the graph of the function.

\[ C = 9.80 + 2.50 \lceil x \rceil \]

<table>
<thead>
<tr>
<th>$x$</th>
<th>$C$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>doesn't make sense</td>
</tr>
<tr>
<td>0.5</td>
<td>$9.80 + 2.50(0.5) = 9.80$</td>
</tr>
<tr>
<td>1</td>
<td>$9.80 + 2.50 = 12.30$</td>
</tr>
<tr>
<td>1.5</td>
<td>$9.80 + 2.50(1) = 12.30$</td>
</tr>
<tr>
<td>2</td>
<td>$9.80 + 2.50(2) = 14.80$</td>
</tr>
</tbody>
</table>

Steps should not be connected.
Definitions: A function $f$ is **even** if, for each $x$ in the domain of $f$, $f(-x) = f(x)$.

A function $f$ is **odd** if, for each $x$ in the domain of $f$, $f(-x) = -f(x)$.

Examples: Determine whether the function is even, odd, or neither.

1. $f(x) = -9$
   
   $f(-x) = -9 = f(x)$ **even**

2. $f(x) = -x^2 - 8$
   
   $f(-x) = -(-x)^2 - 8 = -x^2 - 8 = f(x)$ **even**

3. $g(x) = x^3 - 5x$
   
   $g(-x) = (-x)^3 - 5(-x) = -x^3 + 5x$
   
   $= - (x^3 - 5x) = -f(x)$ **odd**
Example: Find the coordinates of a second point on the graph of a function \( f \) if the given point is on the graph and the function is (a.) even and (b.) odd.

Given point: \((-3, 7)\)

\[- \frac{x}{3} \quad \frac{x}{3} \]

(a.) even \( f(-x) = f(x) \)

\[\Rightarrow f(3) = 7 \quad (3, 7)\]

(b.) odd \( f(-x) = -f(x) \)

\[f(3) = -7 \quad (3, -7)\]