12.4 Expected Value (Expectation)

**Expected value** – expected results of an experiment or business venture over the long run, or,

\[ E = P_1A_1 + P_2A_2 + \ldots + P_nA_n \]

where \( P_i \) = probability that event \( i \) occurs

and \( A_i \) = net amount won or lost if \( i \) occurs

**Fair price:** Expectation = fair price – cost to play

\[ \text{Fair price} = \text{expectation} + \text{cost to play} \]
Examples:

1. **Seminar Attendance.** At an investment tax seminar, Judy Johnson estimates that 20 people will attend if it does not rain and 12 people will attend if it rains. The weather forecast indicates there is a 40% chance it will not rain and a 60% chance it will rain on the day of the seminar. Determine the expected number of people who will attend the seminar.

\[
\begin{array}{ccc}
\text{i} & \text{P}_i & \text{A}_i \\
\text{1 rain} & .6 & 12 \\
\text{2 not rain} & .4 & 20 \\
\end{array}
\]

\[
E = P_1A_1 + P_2A_2 \\
= (.6)(12) + .4(20) = 15.2
\]
2. **Buying Stock**. The Palm Coast investment club is considering purchasing a certain stock. After considerable research, the club members determine that there is a 60% chance of making $8000, a 10% chance of breaking even, and a 30% chance of losing $6200. Find the expectation of this purchase.

<table>
<thead>
<tr>
<th>$A$</th>
<th>$P_A$</th>
<th>$A_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 (make $8000)</td>
<td>.6</td>
<td>8000</td>
</tr>
<tr>
<td>2 (break even)</td>
<td>.1</td>
<td>0</td>
</tr>
<tr>
<td>3 (lose $6200)</td>
<td>.3</td>
<td>-6200</td>
</tr>
</tbody>
</table>

$$E = P_1A_1 + P_2A_2 + P_3A_3$$

$$= .6(8000) + .1(0) + .3(-6200)$$

$$= \text{ } 2940.$$
#22. Raffle Tickets. Ten thousand raffle tickets are sold for $5 each. Four prizes will be awarded – one for $10,000, one for $5,000, and two for $1,000. Sidhardt purchases one of these tickets.

a.) Find the expected value.
b.) Find the fair price of a ticket.

<table>
<thead>
<tr>
<th>i</th>
<th>P_i</th>
<th>A_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 win</td>
<td>$10,000</td>
<td>0.001</td>
</tr>
<tr>
<td>2 win</td>
<td>$5,000</td>
<td>0.001</td>
</tr>
<tr>
<td>3 win</td>
<td>$1,000</td>
<td>0.002</td>
</tr>
<tr>
<td>4 lose</td>
<td>$0</td>
<td>0.9996</td>
</tr>
</tbody>
</table>

\[ E = P_1A_1 + P_2A_2 + P_3A_3 + P_4A_4 \]
\[ = 0.0001(9995) + 0.0001(4995) + 0.0002(995) + 0.9996(-5) \]
\[ E = -3.30 \quad \text{(you can expect to lose $3.30)} \]

Fair price = expectation + cost to play

= $3.30 + 5.00 = $1.70
12.5 Tree Diagrams

**Counting Principle:** If a first experiment can be performed in $M$ distinct ways and a second experiment can be performed in $N$ distinct ways, then the two experiments in that specific order can be performed in $MN$ distinct ways.

(Can be extended to more than 2 experiments in sequence.)

**Sample space** – a list of all possible outcomes of an experiment.

Each individual outcome is called a **sample point** or **simple event**.
Example: Suppose a bag contains 1 Red, 1 Blue, and 1 Green marble. List the sample space in each case:

1. A marble is drawn, its color noted, and it is replaced in the bag. A second marble is drawn and its color noted. (Sampling with replacement).

   Sample space:
   - RR
   - BR
   - GR
   - RB
   - BB
   - GB
   - RG
   - BG
   - GG

   Number of possible events:
   $3 \cdot 3 = 9$
A marble is selected, its color noted, and without replacement, a second marble is drawn and its color noted. (Sampling without replacement)

Sample space

<table>
<thead>
<tr>
<th>1st</th>
<th>2nd</th>
</tr>
</thead>
<tbody>
<tr>
<td>R</td>
<td>B</td>
</tr>
<tr>
<td>B</td>
<td>G</td>
</tr>
<tr>
<td>G</td>
<td>R</td>
</tr>
<tr>
<td>C</td>
<td>B</td>
</tr>
</tbody>
</table>

RB  BR  GR
RC  BG  GB

Number of possible outcomes: \(3 \cdot 2 = 6\)
Examples:

1. Six hardboiled eggs, labeled 1 through 6, are placed in a bag. If you select 3 eggs at random, how many sample points will be in the sample space if the eggs are selected
   a.) with replacement  \[ 6 \times 6 \times 6 = 216 \]
   b.) without replacement  \[ 6 \times 5 \times 4 = 120 \]
2. **Door Prizes.** For door prizes, three different CDs will be awarded to three different people. The first person selected gets to choose between Enya, Mariah Carey, and U2. The second person selected chooses between the two remaining CDs. The third person selected is given the left-over CD.

a.) Determine the number of points in the sample space.

\[3 \times 2 \times 1 = 6\]

b.) Construct a tree diagram and determine the sample space.

```
\(E = \text{Enya}\)
\(M = \text{Mariah Carey}\)
\(U = \text{U2}\)

\text{sample space}

1st \quad 2nd \quad 3rd
\begin{align*}
E & \quad M & \quad U \\
M & \quad U & \quad M \\
E & \quad U & \quad E \\
U & \quad E & \quad M \\
M & \quad E & \quad U \\
U & \quad M & \quad E \\
\end{align*}
```

(sampling without replacement)
Determine the probability that

c.) the Mariah Carey CD is selected first.
\[ P(M \text{ is first}) = \frac{2}{6} = \frac{1}{3} \text{ or } 0.333 \]

d.) the Enya CD is selected first and the Mariah Carey CD is selected last.
\[ P(E \text{ is 1st and } M \text{ is last}) = \frac{1}{6} = \frac{1}{6} = 0.167 \]

e.) The CDs are selected in this order: Mariah Carey, U2, Enya.
\[ P(M\text{U}E) = \frac{1}{6} = 0.167 \]
3. TV Viewing. Below we list the TV shows shown at 7 a.m., 9 a.m., and 10 a.m. in Tampa, Florida in May, 2000.

<table>
<thead>
<tr>
<th>Network</th>
<th>7 a.m.</th>
<th>9 a.m.</th>
<th>10 a.m.</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABC</td>
<td>“Good Morning America”</td>
<td>“Martha Stewart”</td>
<td>“Martin Short”</td>
</tr>
<tr>
<td></td>
<td>A7</td>
<td>A9</td>
<td>A10</td>
</tr>
<tr>
<td>CBS</td>
<td>“This Morning”</td>
<td>“Maury”</td>
<td>“Sally Jessy Raphael”</td>
</tr>
<tr>
<td></td>
<td>C7</td>
<td>C9</td>
<td>C10</td>
</tr>
<tr>
<td>NBC</td>
<td>“Today”</td>
<td>“Later Today”</td>
<td>“Donny &amp; Marie”</td>
</tr>
<tr>
<td></td>
<td>N7</td>
<td>N9</td>
<td>N10</td>
</tr>
</tbody>
</table>

Assume one show will be watched during each time slot.
a.) Determine the number of points in the sample space. \[3 \cdot 3 \cdot 3 = 27\]
b.) Construct a tree diagram and determine the sample space.
Determine the probability that

\[ \frac{1}{27} = 0.0370 \]  (N7-N9-N10)

c.) all three shows on NBC are watched.

d.) “Today” and “Martin Short” are watched.  \( \frac{3}{27} = \frac{1}{9} = 0.111 \)  (N7 and A10)

\( \frac{N7-A9-A10}{N7-C9-A10} \)

\( N7-N9-A10 \)

e.) “Martha Stewart” is not watched.

any sequence not containing A9

\[ \frac{18}{27} = \frac{2}{3} = 0.667 \]

Homework Sections 12.4, 12.5