ICTCM Conference

Exploring Functions, Composition and Inverses Using the TI-83 Plus

Friday, October 31, 2003
Allegheny
10:00-11:45
Rosemont, IL

Roseanne Hofmann
Montgomery County Community College

rhofman@mc3.edu

ICTCM November 1, 2003 1
SET Defaults
When you reset defaults on the TI-83 Plus, all defaults in RAM are restored to the factory settings. Stored data and programs are not changed.
Press MemL key (y key, then √ key) (row 9 column E), choose 7:Reset, then 2:Defaults, then 2:Reset.

ARITHMETIC OF FUNCTIONS
Let \( f(x) = x^2 - 1 \) and \( g(x) = \sqrt{x} \). Find the sum, difference, product and quotient.

What is the domain for \( f(x) \)?
What is the domain for \( g(x) \)?

Domain for \( f(x) + g(x) \) : \( (-\infty, \infty) \)  
Range for \( f(x) \) : \( [-\infty, \infty) \)
Domain of \( f(x)/g(x) \): must exclude 0: \((0, \infty)\)

Domain of \( g(x)/f(x) \): must exclude 1: \([0, 1) \cup (1, \infty)\)

Your turn!

Let \( f(x) = x \) and \( g(x) = \sin x \). Find the sum, difference, product and quotients.

\[ f(g(x)) = f(\sin x) = (\sin x)^2 - 1 \]

What is the domain for \( f(g(x)) \)?

Domain for \( f(g(x)) \) = \([0, \infty)\).

\[ g(f(x)) = g(x^2 - 1) = \sqrt{x^2 - 1} \]

What is the domain for \( g(f(x)) \)?

Domain for \( g(f(x)) \) = \((-\infty, -1] \cap [1, \infty)\)

Does \( y_3(x) = f(g(x)) = x \)? Does \( y_4(x) = g(f(x)) = x \)?

Your turn!

Let \( f(x) = x^2 \) and \( g(x) = \sin x \). Find the composition of functions.
Applications of Composition of Functions

In a medical process known as angioplasty, doctors insert a catheter into a heart vein (through a large peripheral vein), and inflate a small spherical-shaped balloon on the tip of the catheter. Suppose the radius \( r \) of the balloon increases at the constant rate of .5 millimeters per second (mm/sec), find the volume after 5 sec. The balloon is ordinarily inflated to a volume no greater than 400 mm\(^3\). How long would it take to reach 400 mm\(^3\)? (Demana & Waits)

The suggested retail price of a new car is \( p \) dollars. The dealership advertised a factory rebate of $1200 and an 8% discount. (Larson/Hostetler/Edwards)

\( a) \) Write a function \( R \) in terms of \( p \), giving the cost of the car after receiving the rebate from the factory.

\( b) \) Write a function \( S \) in terms of \( p \), giving the cost of the car after receiving the dealership discount.

\( c) \) Form the composite functions \( (R \circ S)(p) \) and \( (S \circ R)(p) \). Interpret each.

\( d) \) Find \( (R \circ S)(18,400) \) and \( (S \circ R)(18,400) \). Which yields the lower cost for the car?

| \( R(p) = p - 1200 \) | \( S(p) = .92p \) | \( R(S(p)) = R(.92p) = .92p - 1200 \) Cost if discount taken before rebate. | \( S(R(p)) = S(p - 1200) = .92(p - 1200) = .92p - 1104 \) Cost if rebate is taken before discount. |

\[ R(p) = p - 1200 \]
\[ S(p) = .92p \]
\[ R(S(p)) = R(.92p) = .92p - 1200 \]
\[ S(R(p)) = S(p - 1200) = .92(p - 1200) = .92p - 1104 \]

**TRANSLATIONS**

Let \( y_1(x) = x^2 \)

Let \( y_1(x-2) \) be a translation along the x-axis and \( y_1(x) - 2 \) be a translation along the y-axis.
Translations viewed as function composition with a linear function.

Let \( y_1(x) = x^2 + 2 \)
let \( y_2(x) = x + 3 \). The composition \( y_1(y_2(x)) \) corresponds to a translation \( y_3 = y_1(x+3) \) or a shift along the x axis 3 units to the left.

\[
\begin{align*}
\quad & y_1(x) = x^2 + 2 \\
\quad & y_2(x) = x + 3 \\
\quad & y_3(x) = y_1(x+3) = x^2 + 2 + 3
\end{align*}
\]

The composition \( y_4(x) = y_2(y_1(x)) = x^2 + 2 + 3 \) corresponds to a vertical shift.

**INVERSE FUNCTIONS**

Discuss the function \( y = f(x) = x^2 + 1 \) and its inverse, if it has one.

\( y = x^2 + 1 \) has no inverse. Why?
If the domain is restricted \( y = (x^2 + 1)(x \geq 0) \), the function has an inverse.
Find the inverse algebraically; check graphically.

\[
\begin{align*}
y &= x^2 + 1 \\
y - 1 &= x^2 \\
\sqrt{y - 1} &= x
\end{align*}
\]

Let \( y = x \)
\[
y = \sqrt{x - 1}
\]

Check with the calculator:

\[
\begin{align*}
\text{Plot1} & \quad \text{Plot2} & \quad \text{Plot3} \\
\sqrt{x^2 + 1} & \quad (x \geq 0) \\
\sqrt{x - 1} &
\end{align*}
\]

Why did \( Y_3 \) graph not include any x's less than 1? Why \( Y_3 \neq Y_4 \)?

\[
\begin{align*}
y_3(x) &= y_1(y_2(x)) = y_1(\sqrt{x - 1}) = (\sqrt{x - 1})^2 + 1 = x - 1 + 1 = x \quad x \geq 1 \\
y_3(x) &= y_2(y_1(x)) = y_2(x^2 + 1) = \sqrt{(x^2 + 1)^2} - 1 = \sqrt{x^2} = x \quad x \geq 0
\end{align*}
\]
The graph of the inverse of a function can also be found using the DrawInv command. DrawInv gives inverse relations as well.

Notice that the TRACE option is not available on a function that is drawn using a DRAW command.

**INVERSES - PARAMETRIC EQUATIONS**

Any function can be written parametrically.

The function \( y(x) = x^2 + 2 \) with its natural domain is not a one to one function. Restrict the domain. We have two choices: \( x \geq 0 \), or \( x \leq 0 \). This gives rise to two one to one functions. Each of these has an inverse. Note that we can restrict the domain of a function using a Boolean expression as part of the definition. While \( y(x) = (x^2 + 2)(x \geq 0) \) has domain all real numbers, its graph suggests that the domain is non-negative real numbers. The function defined here is zero for negative numbers and the graph coincides with the x-axis. The function \( y(x) = (x^2 + 2)/(x \geq 0) \) will be undefined for negative values of \( x \). The grapher will skip those values because a portion of the expression is undefined. See the two TRACE examples below where \( x \) has a negative value. In the one case \( y = 0 \) and in the other \( y \) is undefined.

Continue by defining \( y_2(x) = \sqrt{x - 2} \). Check to see if the inverse is correct by composition of functions. Define \( y_3 = y_1(y_2(x)) \) with STYLE Thick and \( y_4 = y_2(y_1(x)) \). Attempt to graph the functions in a standard window. The response of the calculator at this point is dependent on the mode settings.
A choice of Real vs. $a+bi$ determines the output of functions such as the square root function.

This choice also has implications when using composite functions defined on the y=’s screen. The domain of functions on the y=’s screen is a subset of the real numbers. When the output of the function is complex the grapher skips the point; the function is undefined. In complex mode the output of the inner function may be complex and not an appropriate input to the outer function. This causes an error message: ERR: DATA TYPE.

From a pedagogical point of view this feature of the complex mode can be used to force the student to define domains of functions in exercises on composition.

Back to the exploration of inverses of functions. Restrict the domain of the first function to make it one-one. The second function has a natural domain as a real function. In complex mode it has a domain of all real and a range that includes imaginary numbers. This is not a problem for the grapher. It is a problem when the function is used in a composition of function example. The function Y2 needs to be restricted so its domain is real.

The error arises from the fact that the square root function will output complex values. These are not the proper data types for the composition in the grapher that expects real values of x.

Define the functions as indicated in the screen; do a Zoom ZStandard and ZSquare.

rhofman@mc3.edu
The composition in each case $y_1(y_2(x))$ and $y_2(y_1(x))$ is the identity function as required, however, these identity functions are not equal; check the domains.

$$y_1(y_2(x)) = y_1\left(\sqrt{x-2}\right) = \left(\sqrt{x-2}\right)^2 + 2 = x - 2 + 2 = x \quad x \geq 2$$

$$y_2(y_1(x)) = y_2\left(x^2 + 2\right) = \sqrt{(x^2 + 2)^2} - 2 = \sqrt{x^2} = x \quad x \geq 0$$

**Extension:**

We could have restricted the domain in $y_1(x)$ to non-positive numbers and also gotten a function that was one to one.

**Your Turn:** determine its inverse and check the composition of functions as above. Your answers should match these.

For these graphs determine the appropriate composition. What was your inverse?

**Your Turn:** Try some of the trigonometric functions. Calculate the appropriate domain for the Sine function so that the composite of the Sine and its inverse have an appropriate domain.

Defining $y_1(x) = \sin(x)/(-\pi /2 \leq x)(x \leq \pi /2).$ Zoom ZTRIG gives the following graph.

Trace on this graph to check that it has an error in it. At $x=3, y=0$.

This is the result of a common error in the use of parentheses. An extra set of parentheses is needed in the denominator. The corrected graph for $y_1(x) = \sin(x)/(−\pi /2 \leq x)(x \leq \pi /2))$ is

The domain the $y_1(x)$ function is $-\pi /2 \leq x \leq \pi /2$. Now check the compositions of this function with the $\sin^{-1}$. One will have domain $-\pi /2 \leq x \leq \pi /2$, the other $-1 \leq x \leq 1$. Which is associated with $y_2(y_1(x))$ if $y_2(x) = \sin^{-1}(x)$?

**Extension:** Find the inverse of the secant function. Hint: try $y_1(x) = \cos^{-1}(1/x)$. 