\textbf{Notations and Differentials}

1) $\Delta x$ and $\Delta y$ are delta notations; they represent changes in the variables $x$ and $y$ that occur as we travel along the function:

$$m_{\text{sec}} = \frac{\Delta y}{\Delta x}$$

2) $dx$ and $dy$ are differentials; that is they represent changes in the variables $x$ and $y$ that occur as we travel along the tangent line:

$$m_{\text{sec}} = \frac{dy}{dx}$$

3) if $\Delta x$ is very small, $m_{\text{sec}} \approx m_{\text{tan}}$

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$
3) if $\Delta x$ is very small, $m_{sec} \approx m_{tan}$

$$\frac{\Delta y}{\Delta x} \approx \frac{dy}{dx}$$

$\text{Since } \Delta x = dx$

$$\Delta x \cdot \frac{\Delta y}{\Delta x} \approx \frac{dy}{dx} \cdot dx$$

$$\Delta y \approx dy$$

Formulas:

1) $\Delta y \approx dy$
2) $\Delta y = f(x + \Delta x) - f(x)$
3) $dy = f'(x)dx$
4) Average error or relative error of a quantity

$$\frac{\Delta q}{q} \approx \frac{dq}{q}$$
5) % error of a quantity is equal to the average error multiplied by 100.

**Examples:**

Given \( y = f(x) = x^2 + 5 \)

Find:

a) the differential \( dy \)

\[
\frac{dy}{dx} = 2x
\]

b) the values of \( dx, dy, \Delta y \) if \( x = 3 \) and \( \Delta x = 4 \)

\[
\Delta x = 4
\]

\[
\frac{dy}{dx} = 2(3) = 6
\]

\[
\Delta y = f(x + \Delta x) - f(x) = (3 + 4)^2 + 5 - 14 - 5 = 40
\]

\[
\Delta x = 0.001 = dx
\]

\[
\Delta y = f(3.001) - f(3) = (3.001)^2 + 5 - 14 = 0.006
\]
Examples: The area of a circle is determined using a measured radius of 12 cm. The measuring device for the radius of the circle has a possible maximum error of .03 cm.

   a) Use differentials to estimate the maximum error in the calculated area of the circle

   b) What is the relative error?

\[ A = \pi r^2 \]

\[ dA = 2\pi r \, dr \]

\[ \frac{dA}{A} = \frac{2\pi (12)(.03)}{\pi (12)^2} = .005 \]

\[ .5\% \text{ error.} \]
2) A 12 ft ladder leaning against a wall makes an angle $\theta$ changes with the floor. If the top of the ladder is $h$ feet up the wall, express $h$ in terms of $\theta$ and use differentials to estimate the change in $h$ if $\theta$ changes from 60° to 59°. Round off to three decimal places.

\[ \sin \theta = \frac{h}{12} \]

\[ 12 \sin \theta = h \]

\[ 12 \cos \theta d\theta = dh \]

\[ 12 \cos \frac{\pi}{3} \cdot \left( \frac{-\pi}{180} \right) = dh \]

\[ \approx -0.105 \text{ ft.} \]