Chapter 6

Order of Operations

Definitions:

Algebra - from the Arabic word **al-jabr** meaning "reunion of broken parts"

variables - letters used to represent numbers (x, y, ...)

constants - symbols used to represent specific numbers (2, 1/3, π, ...)


algebraic expression - collection of variables, numbers, parentheses, and operation symbols.
(ex. $2x+5$, $\pi$, $y$,
$\frac{x^2 - \sqrt{5}xy}{3x+2y}$)

To evaluate an expression is to find the value of the expression for a given value of the variable.
Two algebraic expressions joined by an equal sign form an equation.
The solution to an equation is that number or numbers that make an equation true. When we find the solution, we solve the equation.
Order of Operations

1. Perform all operations within parentheses or other grouping symbols.

2. Perform all exponential operations (raising to powers or taking of roots).

3. Perform all multiplications and divisions from left to right.

4. Perform all additions and subtractions from left to right.

Please excuse my dear Aunt Sally.
Ex. (p. 260) Evaluate the expression for the given value(s) of the variable(s).

#10. $x^2$, $x = 9$  \[9^2 = 81\]

#12. $-x^2$, $x = -7$  \[-(7)^2 = -49\]

$-x^2$, $x = 7$  \[-7^2 = -49\]

$(-x)^2$, $x = 7$  \[(-7)^2 = 49\]
#24. \(-x^2 + 4xy, x = 2, y = 3\)
\[-2^2 + 4(2)(3) = -4 + 24 = \boxed{20}\]

#20. \(5x^2 + 7x - 11, x = -1\)
\[-5(-1)^2 + 7(-1) - 11 = 5(1) - 7 - 11 = \boxed{-13}\]

#28. \((x + 3y)^2, x = 4, y = -3\)
\[(4 + 3(-3))^2 = (4 - 9)^2 = (-5)^2 = 25\]
\[(x + 3y)^2, \quad x = 4, \quad y = -3\]
Determine whether the value(s) is (are) a solution to the given equation.

#30. \(4x - 7 = 15, \ x = -2\)

\[
4(-2) - 7 = 15 \\
-8 - 7 \neq 15 \quad \text{(no)}
\]

#34. \(2x^2 - x - 5 = 0, \ x = 3\)

\[
2(3)^2 - 3 - 5 \neq 0 \quad \text{or} \quad 18 - 8 \neq 0 \quad \text{(no)}
\]

#36. \(y = x^2 + 3x - 5, \ x = 1, y = -1\)

\[
-1 \neq 1 + 3(1) - 5 \quad \text{or} \quad 1 + 3 - 5 \neq -1 \quad \text{(yes)}
\]
Linear Equations in One Variable

Terms – parts that are added or subtracted in an expression. (In $3x+2y$, the terms are $3x$ and $2y$)

Numerical coefficients (coefficients) – usually, the number at the beginning of a term.
(The coefficient of $3x$ is 3.)
(In the term $xy$, the coefficient is 1; in the term $-xy$, the coefficient is $-1$.)

Like terms – terms that have the same variables with the same exponents. If terms are not like, they are unlike.

ex. $3x$ and $-5x$ are like terms
$2x^2y$ and $8x^2y$ are like terms but unlike $6xy^2$. 
To simplify an expression means to combine like terms.

A linear (or first degree) equation in one variable is one in which the exponent on the variable is 1.

(ex. $2x + 3 = 9$)

Equivalent equations are equations that have the same solution.

To solve an equation, isolate the variable by using the four properties of equality.

A proportion is a statement of equality between two ratios.
To simplify or combine like terms, use the properties of the real numbers:

- Distributive: \( a(b+c) = ab + ac \)
- Commutative: \( a + b = b + a \)
  \( ab = ba \)

(Note: \( 5 - 3 \neq 3 - 5; \frac{8}{7} \neq \frac{4}{8} \))

- Associative: \( a + (b + c) = (a + b) + c \)
  \( a(b + c) = (ab) + c \)

(Note: \( 8 - (3 - 2) \neq (8 - 3) - 2 \)
  \( 8 - 1 \neq 5 - 2 \)
  \( 7 \neq 3 \))
ex (from p. 271) Combine like terms:

16. \(-9x - 6x = \underline{-15x}\)

28. \(6x + 2y + 8 - 4x - 9y\)

\(= \underline{2x - 7y + 8}\)

Four properties of equality:

Add. If \(a = b\), then \(a + c = b + c\).

Sub. If \(a = b\), then \(a - c = b - c\).

Mult. If \(a = b\), then \(ac = bc, c \neq 0\).

Div. If \(a = b\), then \(\frac{a}{c} = \frac{b}{c}, c \neq 0\).
#40. \[3y - 4 = 11\]
\[3y = 15\]
\[y = 5\]  

#50. \[\frac{x}{3} + 2x = -\frac{2}{5}\]
\[15\left(\frac{x}{3}\right) + 15(2x) = 15\left(-\frac{2}{5}\right)\]
\[5x + 30x = -6\]
\[35x = -6\]
\[x = \frac{-6}{3} = \frac{-6}{35}\]
\(-6/35 \div 3 + 2 \times (-6/35)\)

\(-2/5\)

\(-.4\)
A proportion is a statement of equality between two ratios.

**Ex.** \( \frac{x}{6} = \frac{5}{9} \)

Solve: \( \text{LCM} = 18 \) mult. by 8

\[
\begin{align*}
\frac{3}{18} \left( \frac{x}{6} \right) &= \frac{2}{18} \left( \frac{5}{9} \right) \\
\frac{3x}{3} &= \frac{10}{3} \\
x &= \frac{10}{3}
\end{align*}
\]