Mat 161 Agenda Day 32 04/07/06

Review Conics
9.1 Sequences and Series
Sequences
Factorial Notation
Summation Notation

Homework: 9.1

Quiz on Conics
3. \[ 4y^2 - 2x^2 - 4y - 8x - 15 = 0 \]

\[ 4y^2 - 4y \quad -2x^2 - 8x \quad = 15 \]

\[ 4(y^2 - 1y + \frac{1}{4}) - 2(x^2 + 4x + 4) = 15 + 1 \]

\[ 4(y - \frac{1}{2})^2 - 2(x + 2)^2 = 8 \]

\[ \frac{(y - \frac{1}{2})^2}{2} - \frac{(x + 2)^2}{4} = 1 \]

\[ \frac{1}{2} (-1) = -\frac{1}{2} \]
(a) $\frac{x^2}{144} + \frac{y^2}{169} = 1$

- **Discriminant (D):** Ellipse
- **Center:** (0, 0)
- **Major Vertices:** (6, 13), (6, -13), (0, 13), (0, -13)
- **Asymptotes:** N/A
ID: Ellipse
CONICS
1: CIRCLE
2: ELLIPSE
3: HYPERBOLA
4: PARABOLA

ELLIPSE

\[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

A=13
B=12
H=0
K=0
Find the center and vertices, identify the graph of the equation and sketch the graph:
\[ \frac{x^2}{16} - \frac{y^2}{25} = 1 \]

Identify which conic section:

Characteristics applicable:
Center:
Vertices:
Asymptotes:
$$\frac{(y - \frac{1}{2})^2}{\left(\frac{\sqrt{2}}{2}\right)^2} - \frac{(x + 2)^2}{4^2} = 1$$

**SF:**

**ID:** Hyperbola

**Center:** \((-2, \frac{1}{2})\)

**Vertices:** \((-2, \frac{1}{2} + \sqrt{2})\), \((-2, \frac{1}{2} - \sqrt{2})\)

**Asymptotes**

$$y - \frac{1}{2} = \frac{\sqrt{2}}{2} (x + 2)$$

$$y - \frac{1}{2} = -\frac{\sqrt{2}}{2} (x + 2)$$

$$y = \frac{-\sqrt{2}}{2} x - \frac{1}{2} - \frac{\sqrt{2}}{2}$$

$$y = -\frac{\sqrt{2}}{2} x + \frac{1}{2} + \frac{\sqrt{2}}{2}$$
Complete the square, identify the graph of the equation, identify the characteristics of the conic section and graph: \( 9x^2 + 25y^2 - 36x - 50y + 286 = 0 \)

\[
\begin{align*}
9(x^2 - 4x + 4) + 25(y^2 - 2y + 1) &= -286 + 36 + 25 \\
9(x-2)^2 + 25(y-1)^2 &= -225 \\
\text{positive + positive = no graph.}
\end{align*}
\]
\[
\frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{4} = 1
\]

ELLIPSE

\[
\frac{(X-H)^2}{A^2} + \frac{(Y-K)^2}{B^2} = 1
\]

A = 5
B = 2
H = 2
K = -3

ELLIPSE

\[
\frac{(X-H)^2}{A^2} + \frac{(Y-K)^2}{B^2} = 1
\]

X = 7
Y = -3
Examples: Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola; find the center and vertices.

1. \( x^2 + 4y^2 - 6x + 16y + 21 = 0 \)

\[
\begin{align*}
& \quad \frac{x^2 - 6x + 9}{9} + \frac{4y^2 + 16y + 16}{16} = -21 + 9 \\
& \quad 4(y^2 + 4y + 4) = 4 \\
& \quad \frac{(x-3)^2}{4} + \frac{4(y+2)^2}{4} = \frac{4}{4} \\
& \quad \frac{(x-3)^2}{4} + (y+2)^2 = 1 \\
& \quad \text{Ellipse} \\
& \quad \text{Center (3, -2)} \\
& \quad \text{Major Axis: (3, -1)} \\
& \quad \text{Minor Axis: (3, -3)}
\end{align*}
\]
2. \( y^2 - 4y - 4x = 0 \)

\[ (y-k)^2 = 4f(x-h) \]

SF: \( (y-2) = 4(x+1) \)

ID: Parabola

\[ \theta = 1 \quad x = \hat{a} \]

\[ y = x \quad \int_{-\infty}^{\infty} x = y \]
\[ \frac{1}{a} (y-k) = \frac{a}{2} (x-h)^2 \]

\[ (x-h)^2 = \frac{2a}{9} (y-k) \]

\[ (x-h)^2 = -4p (y-k) \]

\[ (y-k)^2 = 4p (x-h) \]

\[ (y-k)^2 = -4p (x-h) \]
Definitions:
An **infinite sequence** is a function whose domain is the set of positive integers. The function values $a_1, a_2, \ldots, a_n, \ldots$ are the terms of the sequence. If the domain of the function consists of the first $n$ positive integers only, the sequence is a **finite sequence**.

\[
\begin{array}{c|c}
3, 5, 7, 9, \ldots \\
f(x) &= \sqrt{x} \\
\end{array}
\]

\[
\begin{array}{c|c}
x & f(x) \\
0 & 0 \\
\times & \frac{1}{2} \\
\div & \frac{1}{2} \\
\div & \frac{1}{2} \\
1 & \frac{1}{2} \\
\end{array}
\]

\[
\begin{array}{c|c}
n & a_n \\
1 & 3 \\
2 & 5 \\
3 & 7 \\
\end{array}
\]

\[
a_n = 2n + 1
\]
Instead of writing $f(n)$, we write $a_n$. 
Write the first four terms of the following sequences

1. \( a_n = 2n + 5 \)

\[
\begin{array}{c|c}
 n & an \\
\hline
1 & 7 \\
2 & 9 \\
3 & 11 \\
4 & 13 \\
\end{array}
\]

\( 7, 9, 11, 13 \)

\( a_1, a_2, a_3, a_4 \)
\[ a_1 = 2(1) + 5 = 7 \]
\[ a_2 = 2(2) + 5 = 9 \]
\[ a_3 = 2(3) + 5 = 11 \]
\[ a_4 = 2(4) + 5 = 13 \]
2. $b_n = 3^{n-1}$

\[
\begin{align*}
    b_1 &= 3^{1-1} = 3^0 = 1 \\
    b_2 &= 3^{2-1} = 3^1 = 3 \\
    b_3 &= 3^{3-1} = 3^2 = 9 \\
    b_4 &= 3^{4-1} = 3^3 = 27
\end{align*}
\]
\[ b_1 = 3^{1-1} = 1 \]
\[ b_2 = 3^{2-1} = 3 \]
\[ b_3 = 3^{3-1} = 9 \]
\[ b_4 = 3^{4-1} = 27 \]
\[ c_1 = \frac{(-1)^1}{1^2 + 1} = -\frac{1}{2} \]
\[ c_2 = \frac{(-1)^2}{2^2 + 1} = \frac{1}{5} \]
\[ c_3 = \frac{(-1)^3}{3^2 + 1} = -\frac{1}{10} \]
\[ c_4 = \frac{(-1)^4}{4^2 + 1} = \frac{1}{17} \]
If \( n \) is a positive integer, \( n \text{! factorial} \) is defined as
\[
 n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n.
\]
As a special case, zero factorial is defined by \( 0! = 1 \).

\[
\begin{align*}
 5! &= 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \\
 0! &= 1
\end{align*}
\]
What is the difference between $2n!$ and $(2n)!$?

$$2^{(128)} = 240$$

$$2 \cdot 5!$$

$$2n! \quad (2n)!$$

$$2 \cdot n \cdot (n-1) \cdot (n-2) \ldots 1$$

$$(2n) \cdot (2n-1) \cdot (2n-2) \cdot (2n-3) \ldots 1$$
\[ 2n! = 2n(n - 1)(n - 2) \cdots 2 \cdot 1 \]

\[ (2n)! = 2n(2n - 1)(2n - 2) \cdots 2 \cdot 1 \]
Write the first four terms of the following sequence.

\[ a_n = \frac{n^2}{n!} \]

\[ a_1 = \frac{1^2}{1!} = 1 \]

\[ a_2 = \frac{2^2}{2!} = \frac{4}{2} = 2 \]

\[ a_3 = \frac{3^2}{3!} = \frac{9}{3 \cdot 2 \cdot 1} = \frac{3}{2} \]

\[ a_4 = \frac{4^2}{4!} = \frac{16}{4 \cdot 3 \cdot 2 \cdot 1} = \frac{2}{3} \]
\[ a_1 = \frac{1^2}{1!} = 1, \quad a_2 = \frac{2^2}{2!} = 2, \quad a_3 = \frac{3^2}{3!} = \frac{3}{2}, \quad a_4 = \frac{4^2}{4!} = \frac{2}{3} \]
Evaluate the factorial expressions.
a) \( \frac{10!}{2! \cdot 8!} \)
\[
a) \quad \frac{10!}{2! \ 8!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 8!} = \frac{10 \cdot 9}{2} = 5 \cdot 9 = 45
\]
\[ \frac{n!}{(n+1)!} \]
b) \[
\frac{n!}{(n+1)!} = \frac{n!}{(n+1)n!} = \frac{1}{n+1}
\]
3. $c_n = \frac{(-1)^n}{n^2 + 1}$
The sum of the first \( n \) terms of a sequence is represented by

\[
\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n
\]

where \( i \) is called the **index of summation**, \( n \) is the **upper limit of summation**, and 1 is the **lower limit of summation**. This is called a **finite series**.
The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

where $i$ is called the index of summation, $n$ is the upper limit of summation, and 1 is the lower limit of summation. This is called a finite series.

If $n = \infty$, the series is called an infinite series.
We will be concerned only with finite series.
σ is the Greek letter sigma
\[ \sum_{i=1}^{12} x_i \] is called **sigma notation** or **summation notation**.
1. \[ \sum_{i=1}^{n} c = cn, \text{ \(c\) a constant.} \]
2. \( \sum_{t=1}^{n} ca_t = c \sum_{t=1}^{n} a_t \), \( c \) a constant.
3. \[ \sum_{j=1}^{n} (a_j + b_j) = \sum_{j=1}^{n} a_j + \sum_{j=1}^{n} b_j \]
4. \[ \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \]
Examples: Use sigma notation to write the sum.

1. \( \frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15} \)
2. \( \frac{1}{2} - \frac{1}{4} + \frac{1}{8} - \frac{1}{16} + \ldots - \frac{1}{128} \)
3. \[ \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{10 \cdot 12} \]
Examples: Find the sum.

4. \[ \sum_{i=1}^{6} (3i - 1) \]
5. \[ \sum_{j=1}^{5} 6 \]
6. $\sum_{k=2}^{5} (k + 1)(k - 3)$