10.2 Translation of Conics

Homework: 10.1, 10.2
Quiz on Conics Friday
HELP US GET THE WORD OUT!!  Please encourage all students searching for employment (FT, PT, Summer, or Internship) to come to the Student Success Center/Career Services Job Fair this Friday, April 7th from 10 a.m. to 2 p.m. in the Physical Education Building gym. Over 80 employers will be recruiting in all majors; a list may be accessed from the job fair announcement on the college webpage. This is also a good opportunity for students to talk to employers about various careers! Students may call me at (215) 641-6619 with questions. The public is also welcome.

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(a) \[
\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1
\]

(b) \[
\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1
\]

**Major axis is horizontal.**

\[
\sqrt{(x+c)^2 + y^2} + \sqrt{(x-c)^2 + y^2} = 2a
\]

\[
\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1
\]
Definition: An ellipse is the set of all points \((x, y)\) in a plane, the sum of whose distances from two distinct points (foci) is constant.
The line through the foci intersects the ellipse at two points, the vertices. The chord joining the vertices is the major axis and its midpoint is the center of the ellipse. The chord perpendicular to the major axis is the minor axis.
Hyperbola

\[ \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \]
Definition: A **hyperbola** is the set of all points \((x, y)\) in a plane, the difference of whose distances from two distinct fixed points (foci) is a positive constant. The graph of a hyperbola has two disconnected parts (branches). The line through the foci intersects the hyperbola at two points (vertices). The line segment connecting the vertices is the **transverse axis**, and the midpoint of the transverse axis is the **center** of the hyperbola.
The standard form of the equation of a hyperbola with center at the origin (where \( a \neq 0, b \neq 0 \)) is given by

\[
\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{or} \quad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1.
\]
The vertices and foci are $a$ and $c$ units from the center, respectively. Moreover, $a$, $b$, and $c$ are related by the equation $b^2 = c^2 - a^2$. 
To find the asymptotes of a hyperbola, we may use a shortcut.

1. Replace “1” with “0” in the equation.

2. Solve for y. These two equations are the equations of the asymptotes of the hyperbola.
The midpoint between the foci is the center.
The points at which the line segment through the foci meets the hyperbola are the vertices.
The line segment joining the vertices is the transverse axis.
Parabola: Vertex = \((h, k)\)

Directed distance from vertex to focus = \(p\)

\[
(x - h)^2 = 4p(y - k)
\]

Focus: \((h, k + p)\)

Vertex: \((h, k)\)

\[
(y - k)^2 = 4p(x - h)
\]

Focus: \((h + p, k)\)
Parabola:

\[(x - h)^2 = 4p(y - k)\]
\[(y - k)^2 = 4p(x - h)\]

Vertex: \((h, k)\)
Standard Forms of Equations of Conics

**Circle:** Center = \((h, k)\); radius = \(r\)
\[
(x - h)^2 + (y - k)^2 = r^2
\]

**Ellipse:** Center = \((h, k)\)
Major axis length = \(2a\); minor axis length = \(2b\)

\[
\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1
\]

\[
\frac{(x - h)^2}{b^2} + \frac{(y - k)^2}{a^2} = 1
\]
Circle:

\[(x - h)^2 + (y - k)^2 = r\]

Center: \((0, 0)\)   Radius \(r\)

Ellipse:

\[\frac{(x - h)^2}{a^2} + \frac{(y - k)^2}{b^2} = 1\]

Center: \((h, k)\)   Major axis of length \(2a\)
Hyperbola: Center = \((h, k)\)

Transverse axis length = 2\(a\); conjugate axis length = 2\(b\)
Hyperbola:

\[ \frac{(x - h)^2}{a^2} - \frac{(y - k)^2}{b^2} = 1 \]

\[ \frac{(y - k)^2}{a^2} - \frac{(x - h)^2}{b^2} = 1 \]

Center: \((h, k)\) \hspace{1cm} Transverse axis of length \(2a\)
Put in standard form:

2. \( x^2 - 2x - 16y - 31 = 0 \)

\[
(x-1)^2 = 16(y + 3) + 1
\]

\[
(x-h)^2 = 4p(y-k)
\]

\[
\frac{1}{2}(-2) = 1
\]
\[ f = 4 \]

\[
(x - h)^2 = 4p(y - k)
\]

\[
(x - 1)^2 = 16(y + 2)
\]

Identify: parabola

Vertex: \((1, -2)\)

y-intercept: \((0, \frac{3}{16})\)

\[ x^2 - 2x - 16y - 31 = 0 \]

\[-16y - 31 = 0 \]

\[ y = -\frac{31}{16} \]
\[ x^2 - 2x - 16y - 31 = 0 \]

\[ x^2 - 2x = 16y + 31 \]

\[ x^2 - 2x + 1 = 16y + 31 + 1 \]

\[ (x - 1)^2 = 16(y + 2) \]
Put in Standard Form:

3. \[25x^2 + 9y^2 - 200x + 36y + 211 = 0\]

\[\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1\]

\[
\frac{25x^2 - 200x}{25(x^2 - 8x + 16)} + \frac{9y^2 + 36y}{9(y^2 + 4y + 4)} = -211
\]

\[
25(x - 4)^2 + 9(y + 2)^2 = \frac{225}{225}
\]

\[
\frac{(x-4)^2}{\frac{225}{25}} + \frac{(y+2)^2}{\frac{225}{9}} = 1
\]

Id: Ellipse
Center: (4, -2)
Vertices:
CONICS
1: CIRCLE
2: ELLIPSE
3: HYPERBOLA
4: PARABOLA

ELLIPSE

1: \( \frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \)

2: \( \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1 \)

Info | Quit | Esc
\[25x^2 + 9y^2 - 200x + 36y + 211 = 0\]

\[25(x^2 - 8x) + 9(y^2 + 4y) = -211\]

\[25(x^2 - 8x + 16) + 9(y^2 + 4y + 4) = -211 + 400 + 36\]

\[25(x - 4)^2 + 9(y + 2)^2 = 225\]

\[\frac{(x - 4)^2}{9} + \frac{(y + 2)^2}{25} = 1\]
Examples: Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola; find the center and vertices.

1. $x^2 + 4y^2 - 6x + 16y + 21 = 0$
\[(x - 6x + \ldots) + 4(y^2 + 4y + \ldots) = -21\]
\[(x - 6x + 9) + 4(y^2 + 4y + 4) = -21 + 9 + 16\]
\[(x - 3)^2 + 4(y + 2)^2 = 4\]
\[
\frac{(x - 3)^2}{4} + \frac{(y + 2)^2}{1} = 1
\]

Ellips with center \((3, -2)\) and vertices \((1, -2)\) and \((5, -2)\).
2. \[ y^2 - 4y - 4x = 0 \]
3. \[4y^2 - 2x^2 - 4y - 8x - 15 = 0\]
\[ x^2 + 4y = 0 \]
(a) \( \frac{x^2}{144} + \frac{y^2}{169} = 1 \)
Examples: Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola; find the center and vertices.

1. \[ x^2 + 4y^2 - 6x + 16y + 21 = 0 \]
3. \[4y^2 - 2x^2 - 4y - 8x - 15 = 0\]
Find the center and vertices, identify the graph of the equation and sketch the graph:

\[ \frac{x^2}{16} - \frac{y^2}{25} = 1 \]

Identify which conic section: hyperbola

Characteristics applicable:
- Center: \((0, 0)\)
- Vertices: \((4, 0), (-4, 0)\)
- Asymptotes:
  - \(y = \frac{5}{4}x\)
  - \(y = -\frac{5}{4}x\)

\[ \sqrt{\frac{25}{16}} x^2 = \sqrt{y^2} \]

Title: Apr 3 - 9:57 AM (39 of 41)
Complete the square, identify the graph of the equation, identify the characteristics of the conic section and graph: \(9x^2 + 25y^2 - 36x - 50y + 25 = 0\)

**Completed square:**

Identify which Conic Section: **Ellipse**

Characteristics applicable:

Center: \((2, 1)\)

Vertices:

Asymptotes: \(\text{NA}\)

\[
9(x - 4)^2 + 25(y - 2)^2 = 164
\]

\[
\frac{(x - 2)^2}{25} + \frac{(y - 1)^2}{9} = 1
\]
\[
\frac{(x - 2)^2}{25} + \frac{(y + 3)^2}{4} = 1
\]