Equations involving radicals
Equations involving fractions or absolute value

Properties of Inequalities
   Solving a Linear Inequality
   Solving inequalities involving Absolute Value
   Polynomial Inequalities
   Rational Inequalities

Quiz

**Homework:** 2.4, 2.5
and Worksheet
Example: (From page 208.)

131. The distance $d$ (in miles) a car can travel on one tank of fuel is approximated by $d = -0.024s^2 + 1.455s + 431.5; 0 < s \leq 75$, where $s$ is the average speed of the car in miles per hour.
How long did the trip take?

\[ t = \frac{d}{v} = \frac{453}{30} = 15.1 \text{ hours} \]
b. Use the graph to determine the greatest distance that can be traveled on a tank of fuel. How long will the trip take?
\[ y_1 = x^3 - x^2 - 4x + 4 = 0 \]

\[ x^2(\sqrt{x-1}) - 4(\sqrt{x-1}) = 0 \]

\[ (x-1)(x^2-4) = 0 \]
\[ (x-1)(x-2)(x+2) = 0 \]

\[ x = 1, \ x = 2, \ x = -2 \]
3. \[36t^4 + 29t^2 - 7 = 0\]

\[(36t^2 - 7)(t^2 + 1) = 0\]

\[36t^2 - 7 = 0\]

\[t^2 = \frac{7}{36}\]

\[t = \pm \frac{\sqrt{7}}{6}\]

\[t = \pm \sqrt{\frac{7}{6}} + 0.1\]

\[t^2 + 1 = 0\]

\[t^2 = -1\]

\[t = \pm \text{i}\]

\[36 - 7\]

\[1 + 1\]

\[+36\]

\[-7\]

\[+29\]

\[36 - 1\]

\[\times\]
Examples:    Solve.

1.   \(20x^3 - 125x = 0\)

   \[5x(4x^2 - 25) = 0\]

   \[5x(2x + 5)(2x - 5) = 0\]

   \[5x = 0\]
   \[x = 0\]

   \[2x + 5 = 0\]
   \[2x = -5\]
   \[x = \frac{-5}{2}\]

   \[2x - 5 = 0\]
   \[x = \frac{5}{2}\]
6. \[ 4x^2(x-1)^3 + 6x(x-1)^3 = 0 \]

\[ 2x(x-1)^{\frac{1}{3}} \left[ 2x + 3(x-1) \right] = 0 \]

\[ 2x(x-1)^{\frac{1}{3}} \left[ 2x + 3x - 3 \right] = 0 \]

\[ 2x = 0 \quad \text{or} \quad (x-1)^{\frac{1}{3}} = 0 \]

\[ x = 0 \quad \text{or} \quad x = 1 \]
7. \( \sqrt{x+5} = \sqrt{x-5} \)

\[
\left( \sqrt{x+5} \right)^2 = \left( \sqrt{x-5} \right)^2
\]

\( x+5 = x-5 \)

\( \text{Of } x \neq -10, \text{ no solution} \)
8. \(\sqrt{x} + \sqrt{x-20} = 10\)

\[
\sqrt{36} + \sqrt{36-20} = 10 \\
6 + 4 = 10
\]

\[
\left(\sqrt{x-20}\right)^2 = \left(10 - \sqrt{x}\right)^2
\]

\[
x - 20 = 100 - 20\sqrt{x} + x
\]

\[-120 = -20\sqrt{x}
\]

\[
6 = \sqrt{x}
\]

\[
36 = x
\]

\[
(10 - \sqrt{x})(10 - \sqrt{x})
\]

\[
0 - 10\sqrt{x} - 10\sqrt{x} + x
\]

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C. **Equations involving Fractions**
(Start by multiplying each side of the equations by the LCD.)

Solve.
\[
\frac{x}{x-1} - \frac{6}{x} = \frac{2}{1}
\]

\[x \neq 1, \quad x = 0\]

\[
\frac{x(x-1)}{x-1} \cdot \frac{x}{x} = \frac{x(x-16)}{x} = \frac{x(x-2)}{1}
\]

\[
x^2 - 6(x-1) = 2x(x-1)
\]

\[
x^2 - 6x + 6 = 2x^2 - 2x
\]

\[0 = x^2 + 4x - 6\]
\[ 0 = x^2 + 4x - 6 \]

\[ x^2 + 4x + 4 = 6 + 4 \]

\[ (x + 2)^2 = 10 \]

\[ x + 2 = \pm \sqrt{10} \]

\[ x = -2 \pm \sqrt{10} \]

\[ x = -2 + \sqrt{10} \quad -2 - \sqrt{10} \]

\[ 2^2 = 4 \]
Examples: Solve.

9. \( \frac{4x-1}{x} = \frac{3}{1} \quad \Rightarrow \quad x \neq 0 \)

\[
3 = x(4x - 1)
\]

\[
3 = 4x^2 - x
\]

\[
0 = 4x^2 - x - 3
\]

\[
(4x + 3)(x - 1) = 0
\]

\[
x = 1 \quad x = -\frac{3}{4}
\]

\[
4 + 3
\]

\[
1 - 1
\]

\[
-4
\]

\[
+3
\]

\[
-1
\]
10. \[ \frac{4}{x} - \frac{5}{3} = \frac{x}{6} \]
D. Equations Involving Absolute Value
(Remember to consider the quantity inside the absolute value may be positive or negative. Graph the equations to verify your results.)

Solve.

\[ |x^2 - 6| = x \]

\[ |4 - 6| = 2 \]

Case 1

\[ |-2| = 2 \neq -2 \]

\[ x^2 - 6 = x \]

\[ x^2 - x - 6 = 0 \]

\[ (x-3)(x+2) = 0 \]

\[ x = 3 \]

\[ x = -2 \]

\[ \text{OK} \]

Case 2

\[ |9 - 6| \]

\[ 3 = -3 \]

\[ -(x^2 - 6) = x \]

\[ x^2 - 6 = -x \]

\[ x^2 + x - 6 = 0 \]

\[ (x+3)(x-2) = 0 \]

\[ x = -3 \]

\[ x = 2 \]

\[ \text{extraneous} \]
\[ |x^2 - 6| = x \]

\[ |9 - 6| = ? \]
\[ 3 \neq -3 \]

\[ |x^2 - 6| = x \]

\[ |4 - 6| = ? \]
\[ -2 = -2 \]

\[ |1 - 3| = ? \]
\[ 2 \neq -2 \]
Examples: Solve.

11. $|3x+2|=7$
12. \(|x-10| = x^2 - 10x\)
7. \( \frac{6}{x} + \frac{8}{x+5} = 3 \)
8. \( \sqrt{x - 4} = 8 \)
Examples: Use a graphing utility to find any points of intersection.

\[ y = \frac{1}{3} x + 2 \]

9. \[ y = \frac{5}{2} x - 11 \]
\[ y = -x \]

10. \[ y = 2x - x^2 \]
Definitions:
To **solve an inequality in the variable** \( x \) **means to find all values of** \( x \) **for which the inequality is true. These values are solutions of the inequality.**
The set of all points on the real number line that represent the solution set is the **graph of the inequality.**
To solve a linear inequality, use the Properties of Inequalities (page 211) to reduce the inequality to a simpler equivalent inequality. Remember that in multiplying and dividing an inequality:

If $a < b$, then $ac < bc, c > 0$ and $ac > bc, c < 0$

$$
\frac{a}{c} < \frac{b}{c}, c > 0 \quad \text{and} \quad \frac{a}{c} > \frac{b}{c}, c < 0.
$$
<table>
<thead>
<tr>
<th>Algebraic Solution</th>
<th>Graphical Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2 - 6x &gt; 5x + 7$</td>
<td></td>
</tr>
<tr>
<td>$2 - 11x &gt; 7$</td>
<td></td>
</tr>
<tr>
<td>$-11x &gt; 5$</td>
<td></td>
</tr>
<tr>
<td>$x &lt; -\frac{5}{11}$</td>
<td></td>
</tr>
<tr>
<td>The solution set is $(-\infty, -5/11)$</td>
<td></td>
</tr>
</tbody>
</table>
Examples: Solve and graph.

1. $x \geq 5$
2. \[ 0 \leq x \leq \frac{7}{2} \]
3. \[ 2x + 7 < 3 \]
4. \(-2 < 3x + 1 < 10\)
5. \(-8 \leq 1 - 3(x - 2) < 13\)
Let $x$ be a variable or an algebraic expression and let $a$ be a real number such that $a \geq 0$.

1. The solutions of $|x| < a$ are all values of $x$ between $-a$ and $a$.

   $|x| < a$ if and only if $-a < x < a$. 
2. The solutions of $|x| > a$ are all values of $x$ that are less than $-a$ or greater than $a$.

$|x| > a$ if and only if $x < -a$ or $x > a$. 

Examples: Solve and graph.

6. \(|x+6| < 3\)
7. \[|x - 20| \geq 4\]
8. \[ \left| \frac{x-3}{2} \right| \geq 5 \]
Solving a Polynomial Inequality

To determine the test intervals on which the values of a polynomial are entirely negative or entirely positive, use the following steps.

1. Find all real zeros of the polynomial, and arrange the zeros in increasing order. The zeros of a polynomial are its critical numbers.

2. Use the critical numbers to determine the test intervals.

3. Choose one representative $x$-value in each test interval and evaluate the polynomial at that value. If the value of the polynomial is negative, the polynomial will have negative values for every $x$-value in the interval. If the value of the polynomial is positive, the polynomial will have positive values for every $x$-value in the interval.
Solve the following inequalities.

a) \[ x^2 + x - 6 > 0 \]
9. \[ 6(x + 2)(x - 1) > 0 \]
Critical numbers: -2 and 1

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Test value</th>
<th>Sign of polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-∞, -2)</td>
<td>-3</td>
<td>+</td>
</tr>
<tr>
<td>(-2, 1)</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>(1, ∞)</td>
<td>2</td>
<td>+</td>
</tr>
</tbody>
</table>

$6(x + 2)(x - 1) > 0$ when $x < -2$ or when $x > 1$.

```
+++0--------0+++++
-2          1
```
10. $4x^2 - x^4 \leq 0$
Write with 0 on one side of the inequality and with a single fraction on the other side of the inequality. Find the critical numbers for both the numerator and denominator. Proceed as for polynomial inequalities.
11. \[ \frac{5 + 7x}{1 + 2x} < 4 \]
Critical numbers are $x = -1/2$ and 1

<table>
<thead>
<tr>
<th>Intervals</th>
<th>Test value</th>
<th>Sign of polynomial</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(-\infty, -\frac{1}{2})$</td>
<td>-1</td>
<td>-</td>
</tr>
<tr>
<td>$(-\frac{1}{2}, 1)$</td>
<td>0</td>
<td>+</td>
</tr>
<tr>
<td>$(1, \infty)$</td>
<td>2</td>
<td>-</td>
</tr>
</tbody>
</table>

The solution is $(-\infty, -\frac{1}{2})$ and $(1, \infty)$. 
**Scatter Plots and Correlation** (pp. 222 - 223)
A relationship between two variables can be expressed as a collection of ordered pairs.
Graph of such a set of ordered pairs is called a **scatter plot**.
- When $y$ *increases* as $x$ increases, $\leftrightarrow$ **positive correlation**
- When $y$ *decreases* as $x$ increases $\leftrightarrow$ **negative correlation**
Example  Chart below gives the profit for a company for the years 1990 to 1999, where 0 corresponds to 1990 and the profit is in millions of dollars.

<table>
<thead>
<tr>
<th>Year</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>5.1</td>
<td>5.22</td>
<td>5.44</td>
<td>5.56</td>
<td>5.8</td>
<td>5.99</td>
<td>6.22</td>
<td>6.68</td>
<td>6.6</td>
<td>6.77</td>
</tr>
</tbody>
</table>

a) Sketch a scatter plot of the data.
b) Does the data seem to have a negative correlation, a positive correlation, or no correlation?
Fitting a Line to Data (pp. 223 - 226)

- Fit a linear equation to two data points.

Find a linear model for the data given in Example 1 above.
Use the points (1, 5.22) and (6, 6.22), to obtain the linear model

\[ y - 5.22 = \frac{1}{5}(x - 1) \]
\[ y = \frac{1}{5}x + 5.02 \]

Same model if two different data points had been chosen?
Least squares regression line
- The linear model that best fits a given collection of data is called the least squares regression line.
- Use the regression feature of a graphing calculator to find the least squares regression line for a given data set.
- Compare the actual data values with the values given by the model.
- **Correlation coefficient**: labeled $r$, measures how well the model fits the data.
- $r$-values range between $-1$ and $1$; closer $|r|$ is to $1$, the better the model fits the data.
Mat 161 Topics for Test 2

- Solve equations and check answers algebraically or graphically.
- Solve by the Quadratic Formula. Write answer in exact form and using a decimal approximation.
- Multiply and divide complex numbers
- Find all solutions, real and complex, algebraically. Verify using calculator.
  - degree 2 - quadratic
  - degree 4
  - with radicals
  - absolute values
  - cubic roots
- Inequalities
  - linear
  - absolute values
  - rational inequalities
- Regression Equation
  - scatter plot
  - predicting other values
- Application Problem