Mat 161 Day 9 Agenda

Quick Review Sheet
Application Problems
Questions
Review sheets for test 1

Quiz 3
1. Identify the common parent function and the transformation shown in the graph:

a) Parent function: \( y = |x| \)

b) Graphed (transformed function)
$y = |x|$

$y = |x+2|$

$y = |x-2|$
2. Find the standard form of the circle with center \((-3, 2)\) and radius 5.

\[(x-h)^2 + (y-k)^2 = r^2\]

\[x^2 + y^2 = 9\]

\[(x-0)^2 + (y-0)^2 = 3^2\]
2. Find the standard form of the circle with center $(-3, 2)$ and radius 5.

\[(x - h)^2 + (y - k)^2 = r^2\]

\[(x - (-3))^2 + (y - 2)^2 = 25\]

\[\sqrt{(y - 2)^2} = \sqrt{25 - (x + 3)^2}\]

\[y - 2 = \pm \sqrt{25 - (x + 3)^2}\]

\[y = 2 \pm \sqrt{25 - (x + 3)^2}\]

\[= 2 - \sqrt{25 - (x + 3)^2}\]
3. Find the distance between the points \((-3, 2)\) and \((7, -5)\)

\[
d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}
\]

\[
d = \sqrt{(-3 - 7)^2 + (2 - (-5))^2}
\]

\[
d = \sqrt{(-10)^2 + 7^2}
\]

\[
d = \sqrt{100 + 49} = \sqrt{149} \approx 12.2
\]
The distance $d$ between the two points is

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$
4. Factor: \((x^2 + 1)^{-2} - x^2(x^2 + 1)^{-3}\)

\[
\begin{align*}
\left( x^2 + 1 \right)^{-3} & \left[ \left( x^2 + 1 \right)^{-2} \left( -1 \right) \right] \\
\left( x^2 + 1 \right)^{-3} & \left[ x^2 + 1 - x^2 \right] \\
\left( x^2 + 1 \right)^{-3} & \left[ 1 \right] = \frac{1}{(x^2 + 1)^3}
\end{align*}
\]
5. Use the table below to find the percent increase of radio stations with Jazz format.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Radio Stations with Jazz Format</th>
</tr>
</thead>
<tbody>
<tr>
<td>1999</td>
<td>243</td>
</tr>
<tr>
<td>2001</td>
<td>213</td>
</tr>
</tbody>
</table>

\[
\text{Rate of Change} = \frac{\text{New} - \text{Old}}{\text{Old}} = \frac{213 - 243}{243} = \frac{-30}{243} = -0.1234
\]

\[
\frac{4.25 - 3.5}{25} = \frac{0.75}{25} = 0.03 \times 100 = 3\%
\]

\[
\frac{0.25}{4.50} = 0.0556\%
\]
A student’s salary in dollars for collecting phone books is given by $S(x) = 30x + 240$

The amount of withholding for taxes is $W(x) = .20x$, where $x$ is salary. Express withholding as a function of number of phone books collected.

\[
W \circ S(x)
\]

\[
W\left(30x + 240\right) = .2 \left(30x + 240\right)
\]

\[
W(x) = .6x + 48
\]
A company produces a toy for which the variable cost is $12.30 per unit and the fixed costs are $98,000. The toy sells for $17.98. Let \( x \) be the number of units produced and sold.

\[
\text{Profit} = \text{Revenue} - \text{Cost}
\]

\[
P(x) = 17.98x
\]

\[
C(x) = 12.30x + 98,000
\]

\[
P(x) = 5.68x - 98,000
\]

<table>
<thead>
<tr>
<th>( x )</th>
<th>( Y_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>13000</td>
<td>-24150</td>
</tr>
<tr>
<td>14000</td>
<td>-18480</td>
</tr>
<tr>
<td>15000</td>
<td>-12810</td>
</tr>
<tr>
<td>16000</td>
<td>-7140</td>
</tr>
<tr>
<td>17000</td>
<td>-1440</td>
</tr>
<tr>
<td>18000</td>
<td>4240</td>
</tr>
<tr>
<td>19000</td>
<td>9920</td>
</tr>
</tbody>
</table>

\( x = 17000 \)
An open box of maximum volume is to be made from a square piece of material 24 centimeters on a side, by cutting out equal squares from the corners and turning up the sides.

Find the function for Volume of the box.

\[ V = (24 - 2x)^2 \cdot x \]
1. If \( f(x) = x^2 - 12x \), find \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \)
3. Given \( f(x) = x^3 \) \( g(x) = 2x + 1 \)

   a. Find \((g \circ f)(x) = \)

   __________________________________________

   b. What is the domain?

   __________________________________________

   c. Find \((g + f)(x) = \)

   __________________________________________

   d. Find \( g^{-1}(x) = \)

   __________________________________________
Distance Formula, Midpoint, and Equation of a Circle

Given two points in the coordinate plane: \((x_1, y_1), (x_2, y_2)\)

The distance \(d\) between the two points is:

\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}. \]
Find the distance between (2, -5) and (8, 3).

\[ d = \sqrt{(8 - 2)^2 + (3 - (-5))^2} \]
\[ = \sqrt{6^2 + 8^2} \]
\[ = \sqrt{36 + 64} \]
\[ = \sqrt{100} \]
\[ = 10 \]
The midpoint of the line segment joining the two points is
\[\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).\]

Find the midpoint of the line segment joining the points 
(-9, 5) and (4, 2).

\[\left(\frac{-9 + 4}{2}, \frac{5 + 2}{2}\right) = \left(-\frac{5}{2}, \frac{7}{2}\right)\]
Find the standard form of the equation of the circle with center at (2, -5) and radius 4.

\[(x - 2)^2 + (y - (-5))^2 = 4^2\]

or

\[(x - 2)^2 + (y + 5)^2 = 16\]
1. Graph $y = -|x| + 6$
2. Find an equation of a line passing through 
(-1,-3) and parallel to the line 2x + y = 19
3. Given \( f(x) = \begin{cases} \sqrt{-x}, & x \leq 0 \\ 6x, & x > 0 \end{cases} \) find \( f(4) \)
4. If \( f(x) = x^2 - 2x \), find \( \frac{f(x + h) - f(x)}{h} \), \( h \neq 0 \)
5. Find the domain of $f(x) = \sqrt{5 - x}$
6. Graph \( f(x) = [[x+1]] \)
7. Determine where function is increasing, decreasing, and whether it has a relative maximum or relative minimum.
8. Determine whether the functions are odd or even or neither.

a. \( f(x) = 2x^3 + 3x^2 \)

b. \( f(x) = 3x^2 - 6 \)
9. Find an equation of a function that shifts \( f(x) = x^2 \) two units up vertically, three units to the right horizontally.
10. Graph \( g(x) = -x^2 + 2 \) using \( f(x) = x^2 \)
11. Given $f(x) = x + 1$ \hspace{1cm} $g(x) = x + 9$

a. Find $(g \circ f)(x) =$

b. Find $(g + f)(x) =$

c. Find $f^{-1}(x) =$
12. Show \( f(x) = \frac{x}{2} \) and \( g(x) = 2x \) are inverse of each other.
<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>-4</th>
<th>-2</th>
<th>-2</th>
</tr>
</thead>
</table>
2. Use inequality notation to describe

   a. all real numbers less than 3

   b. set of real numbers that are less than 4 and at least –2
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>3.</td>
<td>a. Find the distance between –43 and 16.</td>
</tr>
<tr>
<td></td>
<td>b. Use the absolute notation to describe distance between x and 16 is no more than 5.</td>
</tr>
</tbody>
</table>
4. What is the degree of $12x^3 + 5x^2 + 2$. 
5. Evaluate $6x^2 - 2x$ for $x = 3$. 
6. Identify the property

\[ 5(\frac{1}{5} + x) = 5\left(\frac{1}{5}\right) + 5x \]

\[ 3 + (2 + 7) = (3 + 2) + 7 \]

\[ (7 \times 2) \times 4 = 4 \times (7 \times 2) \]
7. Evaluate $-2x^0 \ y^2$ for $y = 2 \ x = 1$
8. Evaluate \( \frac{4(2)^{-1}}{3^{-2}2} \)
9. Simplify \( \left( \frac{x^{-5} y^2}{z^2} \right)^{-3} \)
10. Simplify

a. \((-3x^2)^2 (-3x^2)^2 (-3x^2)^4\)
b. \( \left( \frac{1}{64} \right)^{3/2} \)
11. Simplify and write answers with no exponents.

\[-\frac{3y^{-2}}{(2y)^{-3}}\]
Examples: Factor completely.

\[ x^2 - x - 6 \]

\[ 2x^2 - 3x - 14 \]

\[ 6x^2 - 29x + 35 \]

To see a review of quadratics factoring, go to the following site:

http://faculty.mc3.edu/rhofman/Flash03/Dec17/menufactor.html
P.4 Rational Expressions

Definitions:
The set of real numbers for which an expression is defined is its domain. Two expressions are equivalent if they yield the same value for all numbers in their domain.

The quotient of two algebraic expressions is a fractional expression.
The quotient of two polynomials is a rational expression.
Domains:
For a polynomial, the domain is all real numbers.
For expressions that contain radicals with even indices, the radicand must be greater than or equal to zero.
For rational expressions, the denominator cannot be equal to zero.

Examples:

\[5x^3 - 2x^2 + 6x - 8\] is a polynomial, the domain is all real numbers.

\[\sqrt{x - 2}, \ x - 2 \geq 0; \text{ the domain is } [2, \infty).\]

\[\frac{x}{x - 1}, \ x - 1 \neq 0, \text{ the domain is } (-\infty, 1) \cup (1, \infty).\]
Find the domain:

\[ 2x^2 - 5x - 2 \]

\[ 6x^2 - 9, \ x > 0 \]

\[ \frac{x + 1}{2x + 1} \]

\[ \sqrt{6 - x} \]
Simplifying Rational Expressions
The key to simplifying rational expressions is that the numerator and denominator should not have any common factors. If the numerator and denominator do have common factors, they can be eliminated by using
\[
\frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{b} = \frac{a}{b} \cdot 1 = \frac{a}{b}
\]
Note that \( \frac{c}{c} = 1 \) only if \( c \neq 0 \).
Reduce: \[ \frac{x^2 - 2x + 1}{x^2 - 1} \]
\[
\frac{x^2 - 2x + 1}{x^2 - 1} = \frac{(x - 1)^2}{(x + 1)(x - 1)} = \frac{x - 1}{x + 1} \cdot \frac{x - 1}{x - 1} = \frac{x - 1}{x + 1}, x \neq 1
\]
Reduce:

\[
\frac{x^2 + 8x - 20}{x^2 + 11x + 10}
\]

\[
\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}
\]
Simplify: \( \frac{x^3 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} \)
\[
\frac{x^2 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} = \frac{(x+1)(x^2 - x + 1)}{(x-3)^2} \cdot \frac{(x+3)(x-3)}{x^2 - x + 1}
\]

\[
= \frac{(x+1)(x^2 - x + 1)(x+3)(x-3)}{(x-3)^2 (x^2 - x + 1)}
\]

\[
= \frac{(x+1)(x+3)}{(x-3)} \cdot \frac{x^2 - x + 1}{x^2 - x + 1}
\]

\[
= \frac{(x+1)(x+3)}{(x-3)} \cdot 1, x \neq 3
\]

\[
= \frac{(x+1)(x+3)}{(x-3)}, x \neq 3
\]
Simplify: \( \frac{x+6}{x} + \frac{2}{x^2 + x} \)
\[
\frac{x+6}{x} + \frac{2}{x^2+x} = \frac{x+6}{x} \cdot \frac{x+1}{x+1} + \frac{2}{x(x+1)}
\]

\[
= \frac{(x^2 + 7x + 6) + 2}{x(x+1)}
\]

\[
= \frac{x^2 + 7x + 8}{x(x+1)}
\]
Complex Fractions:
Fractions that contain fractions in the numerator or denominator are called complex fractions. There are two ways to simplify a complex fraction. Here are examples of each.
Order of Operations:

\[
1 - \frac{\frac{2}{x}}{\frac{5}{x-1} + \frac{4}{x}} = \frac{x - 2}{x(\frac{9x - 4}{x(x - 1)})}
\]

\[
= \frac{x - 2}{x} \cdot \frac{x(x - 1)}{9x - 4}
\]

\[
= \frac{(x - 2)(x - 1)}{9x - 4}, \quad x \neq 0
\]
Multiply by the LCD.

\[
\frac{1 - \frac{2}{x}}{\frac{5}{x - 1} + \frac{4}{x}} = \frac{1 - \frac{2}{x}}{\frac{5}{x - 1} + \frac{4}{x}} \cdot \frac{x(x - 1)}{x(x - 1)}
\]

\[
= \frac{x(x - 1) - 2(x - 1)}{5x + 4(x - 1)}
\]

\[
= \frac{x^2 - 3x + 2}{9x - 4}
\]
Rationalize the numerator: \( \frac{\sqrt{x} + 1 - 1}{x} \)
\[
\frac{\sqrt{x+1}-1}{x} = \frac{\sqrt{x+1}-1}{x} \cdot \frac{\sqrt{x+1}+1}{\sqrt{x+1}+1}
\]
\[
= \frac{(\sqrt{x+1})^2 - 1^2}{x(\sqrt{x+1}+1)}
\]
\[
= \frac{x}{x(\sqrt{x+1}+1)}
\]
\[
= \frac{1}{\sqrt{x+1}+1}, \ x \neq 0
\]
Perform the operations and simplify:

\[
\frac{4y-16}{5y+15} \cdot \frac{2y+6}{4-y}
\]
\[
\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8}
\]
\frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{x^2-1}
\[\frac{x - 4}{x - 4} \cdot \frac{4}{x}\]
$5x^5 - 3x^{\frac{3}{2}}$
\[ 2x(x - 5)^{-3} - 4x^2(x - 5)^{-4} \]