Collect Quiz

Review of Unit 4 materials

**Homework:** Study for Test 4

**Test 4 Friday**
Find the sum \[ \sum_{k=2}^{6} (-1)^k (2k) \]
Problem 1: A company has a fixed monthly manufacturing cost of $12,000, and it costs $0.95 to produce each unit. The company then sells each unit for $1.25. How many units must be sold before this company breaks even?

C = 12,000 + 0.95x and
R = 1.25x. For the break-even point, C = R.

\[
\text{Profit} = 0
\]

\[
\text{Profit} = \text{Revenue} - \text{Cost} = 0
\]

\[
\text{Revenue} = \text{Cost}
\]

\[
1.25x = 12,000 + 0.95x
\]

\[
0.30x = 12,000
\]

\[
x = \frac{12,000}{0.30} = 40,000 \text{ units}
\]

\[
R = \$50,000
\]
Problem 2: On a Saturday night, the manager of a shoe store evaluates the receipts of the previous week's sales. Two hundred forty pairs of two different types of tennis shoes were sold. One style sold for $66.95 and the other sold for $84.95. The total receipts for the week were $17,652. The cash register was supposed to record the number of each type of shoe sold but it malfunctioned; help the manager determine how many of each type of shoe were sold.

Let \( x = \# \) type A shoe (10s)
\( y = \# \) type B shoe (100s)

\( x + y = 240 \)
\( 66.95x + 84.95y = 17,652 \)

\[
\begin{bmatrix}
1 & 152 \\
0 & 188 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

\( x = 152 \)  
\( y = 88 \)  

3 unknowns 2 equations

\[
\begin{bmatrix}
1 & 1.5 & 6 \\
0 & 1 & 16 \\
\end{bmatrix}
\rightarrow
\begin{bmatrix}
1 & 0 & 3 \\
0 & 1 & 4 \\
\end{bmatrix}
\]
Problem 3: Choice of Two Jobs - You are offered two different jobs selling college textbooks. TextsNotDotCom offers an annual salary of $25,000 plus a year end bonus of 1% of your total sales. Cheapbooks offers an annual salary of $20,000 plus a year end bonus of 2% of your total sales. Find the annual sales that make Cheapbooks the better offer.

\[
W = 25,000 + 0.01 S \\
W_c = 20,000 + 0.02 S \\
25,000 + 0.01 S = 20,000 + 0.02 S \\
5,000 = 0.01 S \\
\$500,000 = S
\]
Problem 1: A man in a boat can row 8 miles downstream in 1 hour. He can row 6 miles upstream in 3 hours. How fast can the man row in still water and what is the rate of current?

\[ \text{Distance} = \text{Rate} \times \text{Time} \]

\[ 8 = (r + c) \times 1 \]
\[ 6 = (r - c) \times 3 \]

\[ \frac{8}{2} = r + c \]
\[ 2 = r - c \]

\[ \frac{10}{2r} = 5 = r \]
\[ 3 = c \]
\[
\begin{align*}
\begin{cases}
r + c &= 8 \\
3(r - c) &= 6 \\
3r + 3c &= 24 \\
3r - 3c &= 6
\end{cases}
\end{align*}
\]

\[6r = 30 \Rightarrow r = 5 \Rightarrow c = 3\]

The man can row 5 mph in still water. The rate of current is 3 mph.
Problem 2: You have $10,000 to invest in two simple interest funds. One pays 8% and the other 6%. How much should be invested in each so that the total annual interest is $720?

Let $x = \text{amt} @ 8\%$

$y = \text{amt} @ 6\%$

$x + y = 10,000$

$0.08x + 0.06y = 720$
\[
\begin{align*}
\{ & x + y = 10,000 \\
& 0.08x + 0.06y = 720 \Rightarrow -8x - 8y = -80,000 \\
& 8x + 6y = 72,000 \\
& -2y = -8,000 \\
& y = 4,000 \Rightarrow x = 6,000
\end{align*}
\]

You should invest $6,000 at 8% and $4,000 at 6%.
\[ \sum_{n=3}^{6} \frac{3}{n - 2} \]
2. Write in sigma notation
\[ \frac{2}{1} + \frac{3}{2} + \frac{4}{3} + \ldots + \frac{7}{6} \]
Solve \( x^2 + y^2 = 25 \)
\( x - y = 1 \)
Solve \( x^2 + y^2 = 25 \)
\[ x - y = 1 \]

\[ \Rightarrow x = 1 + y \]
\[ \Rightarrow y^2 = 25 - x^2 \]

\[ y_1 = \sqrt{25 - x^2} \]
\[ y_2 = -\sqrt{25 - x^2} \]

\[ (1 + y)^2 + y^2 = 25 \]
\[ 1 + 2y + y^2 + y^2 = 25 \]
\[ 2y^2 + 2y - 24 = 0 \]
\[ 2(y^2 + y - 12) = 0 \]
\[ (y + 4)(y - 3) = 0 \]

\[ y + 4 = 0 \quad y - 3 = 0 \]
\[ y = -4 \quad y = 3 \]
\[ x = 1 + y = -3 \quad x = 4 \]

\[ (4, 3) \quad (-3, 4) \]
Solve by substitution
6x + 2y = 7
4x - 7y = -37
5. Solve by elimination

\[ \frac{6}{x} + \frac{1}{y} = -2 \]
\[ \frac{4}{x} - \frac{3}{y} = 17 \]
Solve for x: 
\[x + 2.5y = 900\]
\[5x - 2y = 150\]
7. Solve for x, y, z
\[ x + 2y - z = 26 \]
\[ y + 3z = 5 \]
\[ z = -2 \]
8. a. Solve for $x$, $y$, $z$

$x - y + z = 5$
$3x + 2y - z = -2$
$2x + y + 3z = 10$
\[ x + y + z = 4 \]
\[ x - 3y - z = 1 \]
\[ 2x - 2y = 9 \]
9. Find an equation

\[ y = ax^2 + bx + c \]

that passes through \((0, 5)\), \((2, -5)\), \((-3, -40)\).

\[
\begin{align*}
5 &= a(0)^2 + b(0) + c \\
-5 &= a(2)^2 + b(2) + c \\
-40 &= a(-3)^2 + b(-3) + c
\end{align*}
\]

\[
\begin{bmatrix}
0 & 0 & 5 \\
4 & 2 & -5 \\
9 & -3 & 1 \end{bmatrix}
\]

\[
\begin{align*}
c &= 5 \\
4a + 2b + c &= -5 \\
9a - 3b + c &= -40
\end{align*}
\]

\[
y = -4x^2 + 3x + 5
\]
10. Solve by matrices:

\[
\begin{align*}
  x + y + z &= 4 \\
  2x + y + z &= 6 \\
  x + y + 2z &= 9
\end{align*}
\]
11. Classify the graph of $3x^2 + 3y^2 - 6x + 18y + 10 = 0$.
Put in recognizable form.
Give the vertices, asymptotes, center, intercepts where applicable.

\[3x^2 - 6x + 3y^2 + 18y = -10\]

\[3(x-2)^2 + 3(y+3)^2 = 20\]

\[\frac{(x-1)^2}{\frac{20}{3}} + \frac{(y+3)^2}{\frac{20}{3}} = 1\]

\[\sqrt{\frac{20}{3}} \sqrt{\frac{20}{3}} = \frac{\sqrt{600}}{3} = \frac{2\sqrt{15}}{3}\]
14. Graph: $x^2 + 5y^2 = 5$. Put in recognizable form. Identify the graph, give the vertices, asymptotes, center, intercepts where applicable.
15. Put in standard form:
\[ x^2 + 4x - 8y + 4 = 0. \]
Identify the graph, give the vertices, asymptotes, center, intercepts where applicable.

\[
(x - h)^2 = 4p(y - k)
\]
\[
(y - k)^2 = 4p(x - h)
\]

\[ x^2 + 4x + 4 = 8y \]
\[
(x + 2)^2 = 8(y - 0)
\]

Vertex: \((-2, 0)\)

\(-x\) - intercept: \(-2\)

\(y\) - intercept: \((0, 2)\)
16. What is the coefficient of the $x^2y^7$ term in the expansion of $(7x - 2y)^9$?

\[\binom{9}{7} \cdot (7x)^2 \cdot (-2y)^7 \]
\[= \binom{9}{7} \cdot (36) \cdot (49x^2) \cdot (-128y^7) \]
\[= 225792 \cdot x^2y^7 \]
\[
\begin{align*}
7x - 3y + 2w &= 4, \\
-2x + y - w &= -13, \\
4x - y &= 12, \\
-x + y - w &= -8.
\end{align*}
\]

Reduced row echelon form:
\[
\begin{bmatrix}
1 & 0 & 0 & 6 \\
0 & 1 & 0 & 5 \\
0 & 0 & 1 & -2 \\
0 & 0 & 0 & 0
\end{bmatrix}
\]

Since \( \text{rref}(A) \) has a row of zeros, the system is consistent:
\[
\begin{bmatrix}
1 & 0 & 0 & | & 0 \\
0 & 1 & 0 & | & 0 \\
0 & 0 & 1 & | & 0
\end{bmatrix}
\]

Thus, the solution is \( \begin{bmatrix} x \\ y \\ w \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \).
17. Expand $(x - 4)^4$
18. Use the binomial theorem to expand and simplify \((3x + y)^4\)