Review Conics
9.1 Sequences and Series
Sequences
Factorial Notation
Summation Notation

Worksheet on Conic Sections

Homework: 9.1
Quiz on Friday on Conics
Put in standard form:

1. \(4x^2 - 9y^2 - 24x - 72y - 72 = 0\)

\[
4x^2 - 9y^2 - 24x - 72y = 72
\]

\[
4(x^2 - 6x) - 9(y^2 + 8y) = 72
\]

\[
4(x^2 - 6x + 9) - 9(y^2 + 8y + 16) = 72 + 36 - 144
\]

\[
4(x - 3)^2 - 9(y + 4)^2 = -36
\]

\[
\frac{(y + 4)^2}{-3} - \frac{(x - 3)^2}{4} = 1
\]

**Hyperbola**

**Center:** \((3, -4)\)

**Vertices:** \((3, -2), (3, -6)\)

**Asymptotes:**

\[
y + 4 = \pm \frac{2}{3} (x - 3)
\]
Put in standard form:

2. \(x^2 - 2x - 16y - 31 = 0\)

\[
\begin{align*}
(x-1)^2 &= 16(y+3) \\
(x-1)^2 &= 16(y+2) \\
(x-h)^2 &= 4p(y-k) \\
\text{Parabola} \\
\text{Vertex: (1,-2)}
\end{align*}
\]
Put in Standard Form:
3. \(25x^2 + 9y^2 - 200x + 36y + 211 = 0\)

\[
25(x^2 - 8x + 16) + 9(y^2 + 4y + 4) = -211 + 400 + 36
\]

\[
\frac{25(x-4)^2}{225} + \frac{9(y+2)^2}{225} = 1
\]

Ellipse

Center \((4, -2)\)

Vertices:

\((1, -2)\)

\((7, -2)\)

\((4, 3)\)
Examples: Classify the graph of the equation as a circle, a parabola, an ellipse, or a hyperbola; find the center and vertices.

1. \(x^2 + 4y^2 - 6x + 16y + 21 = 0\)
2. \[ y^2 - 4y - 4x = 0 \]
3. \[ 4y^2 - 2x^2 - 4y - 8x - 15 = 0 \]
(a) \( \frac{x^2}{144} + \frac{y^2}{169} = 1 \)
Definitions:
An **infinite sequence** is a function whose domain is the set of positive integers.
The function values \( a_1, a_2, \ldots, a_n, \ldots \) are the **terms** of the sequence.
If the domain of the function consists of the first \( n \) positive integers only, the sequence is a **finite sequence**.

\[
\begin{align*}
f(x) &= 2x + 5 \\
f(1) &= 2(1) + 5 = 7.2
\end{align*}
\]

\[
\begin{array}{c|c}
\hline
n & a_n \\
\hline
1 & 7 \\
2 & 9 \\
3 & 11 \\
\hline
\end{array}
\]

\( n = 1 \times 3 = 7, 9, 11 \)
The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

where $i$ is called the index of summation, $n$ is the upper limit of summation, and 1 is the lower limit of summation. This is called a finite series.

The sum $\sum_{i=1}^{5} \chi_i = \chi_1 + \chi_2 + \chi_3 + \chi_4 + \chi_5$ is equal to $20 + 21 + 25 + 27 + 32$. 

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The sum of the first $n$ terms of a sequence is represented by

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots + a_n$$

where $i$ is called the index of summation, $n$ is the upper limit of summation, and 1 is the lower limit of summation. This is called a finite series.

If $n = \infty$, the series is called an infinite series.
\[ \sum \text{ is the Greek letter sigma} \]
\[ \sum_{i=1}^{n} x_i \] is called **sigma notation** or **summation notation**.
Instead of writing \( f(n) \), we write \( a_n \).

\[
f'(x) = 2x + 5
\]

\[
a_n = 2n + 5
\]
Write the first four terms of the following sequences

1. \( a_n = 2n + 5 \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>( a_n )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>13</td>
</tr>
</tbody>
</table>

\[ \{7, 9, 11, 13\} \]

sequence
\[ a_1 = 2(1) + 5 = 7 \]
\[ a_2 = 2(2) + 5 = 9 \]
\[ a_3 = 2(3) + 5 = 11 \]
\[ a_4 = 2(4) + 5 = 13 \]
2. $b_n = 3^{n-1}$

\[
\begin{array}{c|c|c}
\hline
n & n-1 & 3^{n-1} \\
\hline
1 & 0 & 1 \\
2 & 1 & 3 \\
3 & 2 & 9 \\
4 & 3 & 27 \\
\hline
\end{array}
\]

$\{1, 3, 9, 27\}$
\[ b_1 = 3^1 - 1 = 2 \]
\[ b_2 = 3^2 - 1 = 8 \]
\[ b_3 = 3^3 - 1 = 26 \]
\[ b_4 = 3^4 - 1 = 80 \]
3. \( c_n = \frac{(-1)^n}{n^2 + 1} \)

<table>
<thead>
<tr>
<th>( n )</th>
<th>((-1)^n)</th>
<th>( n^2 + 1 )</th>
<th>(-\frac{1}{n^2+1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-1</td>
<td>2</td>
<td>(-\frac{1}{2})</td>
</tr>
<tr>
<td>2</td>
<td>((-1)^2 = +1)</td>
<td>5</td>
<td>(-\frac{1}{5})</td>
</tr>
<tr>
<td>3</td>
<td>((-1)^3 = -1)</td>
<td>10</td>
<td>(-\frac{1}{10})</td>
</tr>
<tr>
<td>4</td>
<td>((-1)^4 = +1)</td>
<td>17</td>
<td>(-\frac{1}{17})</td>
</tr>
</tbody>
</table>
\[ c_1 = \frac{(-1)^1}{1^2 + 1} = -\frac{1}{2} \]
\[ c_2 = \frac{(-1)^2}{2^2 + 1} = \frac{1}{5} \]
\[ c_3 = \frac{(-1)^3}{3^2 + 1} = -\frac{1}{10} \]
\[ c_4 = \frac{(-1)^4}{4^2 + 1} = \frac{1}{17} \]
If \( n \) is a positive integer, \( n \text{ factorial} \) is defined as \( n! = 1 \cdot 2 \cdot 3 \cdots (n-1) \cdot n \).

As a special case, zero factorial is defined by \( 0! = 1 \).

\[ 0! = 1 \]

\[ 5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120 \]

\[ 2! = 2 \cdot 1 = 2 \]

\[ 3! = 3 \cdot 2 \cdot 1 = 6 \]
What is the difference between $2n!$ and $(2n)!$?

\[
2 \cdot n (n-1)(n-2) \ldots 1
\]

\[
2 \cdot 5 !
\]

\[
2 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1
\]

\[
240
\]

\[
(2n)(2n-1)(2n-2) \ldots 1
\]

\[
10 \cdot 9 \cdot 8 \ldots 1
\]
\[ 2n! = 2n(n - 1)(n - 2) \cdots 2 \cdot 1 \]
\[ (2n)! = 2n(2n - 1)(2n - 2) \cdots 2 \cdot 1 \]
Write the first four terms of the following sequence.

\[ a_n = \frac{n^2}{n!} = \frac{n \cdot n}{n \cdot (n-1) \cdot (n-2) \cdot 1} \]

<table>
<thead>
<tr>
<th>n</th>
<th>(\frac{n^2}{n!})</th>
<th>(a_n)</th>
<th>(\frac{4}{2 \cdot 1})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{1}{1!})</td>
<td>(1)</td>
<td>(\frac{4}{2 \cdot 1})</td>
</tr>
<tr>
<td>2</td>
<td>(\frac{4}{2!})</td>
<td>(\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{2}{1})</td>
<td>(= \frac{3}{2})</td>
</tr>
<tr>
<td>3</td>
<td>(\frac{9}{3!})</td>
<td>(\frac{2}{3} \cdot \frac{4}{3} \cdot \frac{2}{1})</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>(\frac{16}{4!})</td>
<td>(\frac{3}{4} \cdot \frac{3}{2} \cdot \frac{2}{1})</td>
<td></td>
</tr>
</tbody>
</table>
\[ a_1 = \frac{1^2}{1!} = 1, \quad a_2 = \frac{2^2}{2!} = 2, \quad a_3 = \frac{3^2}{3!} = \frac{3}{2}, \quad a_4 = \frac{4^2}{4!} = \frac{2}{3} \]
Evaluate the factorial expressions.
\[
a) \quad \frac{10!}{2! \cdot 8!} = \frac{10 \cdot 9 \cdot 8!}{2 \cdot 8!} = \frac{10 \cdot 9}{2} = 5 \cdot 9 = 45
\]
\[
\frac{n!}{(n+1)!} = \frac{\cancel{n!}}{(n+1) \cancel{n!}} = \frac{1}{n+1}
\]
b) \[
\frac{n!}{(n+1)!} = \frac{n!}{(n+1)n!} = \frac{1}{n+1}
\]
We will be concerned only with finite series.
1. \[ \sum_{i=1}^{n} c = cn, \text{ } c \text{ a constant.} \]

\[
\sum_{i=1}^{5} 6 = 6 + 6 + 6 + 6 + 6 = 30
\]
2. \( \sum_{i=1}^{n} ca_i = c \sum_{i=1}^{n} a_i, \) where \( c \) is a constant.

\[
\sum_{i=1}^{3} 2x_i = 2x_1 + 2x_2 + 2x_3
\]

\[
2 \left( \sum_{i=1}^{3} x_i \right)
\]
3. \[ \sum_{i=1}^{n} (a_i + b_i) = \sum_{i=1}^{n} a_i + \sum_{i=1}^{n} b_i \]

\[
\sum_{i=1}^{3} x_i + y_i
\]

\[
\sum_{i=1}^{3} x_i + y_i = x_1 + y_1 + x_2 + y_2 + x_3 + y_3
\]

\[
\sum_{i=1}^{3} x_i = x_1 + x_2 + x_3
\]

\[
\sum_{i=1}^{3} y_i = y_1 + y_2 + y_3
\]
4. \[ \sum_{i=1}^{n} (a_i - b_i) = \sum_{i=1}^{n} a_i - \sum_{i=1}^{n} b_i \]
Examples: Use sigma notation to write the sum.

1. \[ \frac{5}{1+1} + \frac{5}{1+2} + \frac{5}{1+3} + \ldots + \frac{5}{1+15} \]
\[ \frac{5}{2} + \frac{5}{3} + \frac{5}{4} \]
\[ \sum_{j=1}^{15} \frac{5}{1+j} \]
2. \[ 1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots - \frac{1}{128} \]

\[ \sum_{i=1}^{8} \frac{(-1)^{i+1}}{2^{i-1}} \]

\[ C_5^2 = \frac{5!}{2!3!} = \frac{5 \cdot 4 \cdot 3}{2 \cdot 1 \cdot 3} = 10 \]

\[ n \binom{r}{r} = \frac{n!}{r!(n-r)!} \]
3. \[ \frac{1}{1 \cdot 3} + \frac{1}{2 \cdot 4} + \frac{1}{3 \cdot 5} + \ldots + \frac{1}{10 \cdot 12} \]
Examples: Find the sum.

4. \[ \sum_{i=1}^{6} (3i - 1) \]
5. \[ \sum_{i=1}^{5} 6 \]
6. \[ \sum_{k=2}^{5} (k + 1)(k - 3) \]
Find the center and vertices, identify the graph of the equation and sketch the graph:

\[
\frac{x^2}{16} - \frac{y^2}{25} = 1
\]

Identify which conic section:

- Characteristics applicable:
  - Center:
  - Vertices:
  - Asymptotes:

Due to the nature of the document, the specifics of the conic section characteristics and graph details cannot be transcribed accurately.
Complete the square, identify the graph of the equation, identify the characteristics of the conic section and graph:

\[ 9x^2 + 25y^2 - 36x - 50y + 286 = 0 \]

Completed square:

Identify which Conic Section:

Characteristics applicable:

Center:

Vertices:

Asymptotes: