Review Sheet #21

Quadratic Equations
Polynomial Equations of higher degree
Equations involving radicals
Equations involving fractions or absolute value

Quiz on 2.1, 2.2, 2.3

Homework: 2.4
\[
\left( \frac{2+i}{2-i} \right) \left( \frac{2+i}{2+i} \right) = \frac{4+2i+2i-i^2}{5} = \frac{3+4i}{5} - \frac{1}{2} \cdot \frac{1}{5} + \frac{4}{5}i.
\]

4 + 1
1. Solve the equation and check your answer either algebraically or graphically.

\[
\frac{5}{x+1} - \frac{1}{x-5} = 0
\]

\[
\frac{5}{(x+1)(x-5)} - \frac{1}{(x+1)(x-5)} = 0
\]

\[
5(x-5) - 1(x+1) = 0
\]

\[
5x - 25 - x - 1 = 0
\]

\[
4x - 26 = 0
\]

\[
4x = 26
\]

\[
\frac{4x}{4} = \frac{26}{4}
\]

\[
x = \frac{13}{2}
\]

\[
x = 6.5
\]

Check:

\[
\frac{5}{7.5} - \frac{1}{1.5} = 0
\]

\[
.667 - .667 = 0
\]

\[
x \neq -1, \ x \neq 5
\]
2. A company has fixed costs of $35,000 per month and a variable cost of $17.65 per TV. How many units can be produced if the total cost can not exceed $250,000?

\[
\text{Total Cost} = \text{Fixed Cost} + \text{Variable Cost}
\]

\[
35,000 + 17.65x \leq 250,000
\]

\[
17.65x \leq 250,000 - 35,000
\]

\[
17.65x \leq 215,000
\]

\[
x \leq 12,181 + V
\]
3. Find the x and y intercepts of the graph of the equation: \( xy - 3y + 2x - 1 = 0 \).

\[
(0)\ y - 3y + 2(0) - 1 = 0 \\
-3y - 1 = 0 \\
\frac{-3y}{-3} = \frac{-1}{-3} \\
(0, \frac{1}{3})
\]

\[
x(0) - 3(0) + 2x - 1 = 0 \\
2x = 1 \\
\frac{2x}{2} = \frac{1}{2} \\
(1, 0)
\]

\[
xy - 3y + 2x - 1 = 0 \\
x - 3y + 2x - 1 = 0 \\
x - 3y = -2x + 1 \\
x = \frac{-2x + 1}{y - 3} \\
y = \frac{y - 3}{-2x + 1}
\]

\[
\frac{y}{x-3} = \frac{-2x+1}{x-3} \\
y = \frac{-2x+1}{x-3}
\]
10. \((3 + \sqrt{-5})(7 - \sqrt{-10})\)

\[
\begin{align*}
(3 + \sqrt{5}i) & \cdot (7 - \sqrt{10}i) \\
21 & -3\sqrt{10}i + 7\sqrt{5}i - \sqrt{50}i^2 \\
21 + 5\sqrt{2} + (-3\sqrt{10} + 7\sqrt{5})i & + \sqrt{50}i \\
& \quad 5\sqrt{2}
\end{align*}
\]
8. \[ \frac{2 + 3i}{4 - 2i} \cdot \frac{4 + 2i}{4 + 2i} \]

\[
\frac{8 + 4i + 12i + 6i^2}{16 - 4i^2} = \frac{2 + 16i}{20} \cdot \frac{1}{10} + \frac{4}{5}i
\]
\[
\frac{1}{1+i} \cdot \frac{1-i}{1-i} = \frac{1-i}{1-i^2} = \frac{1-i}{2}
\]

\[
\frac{1}{2} - \frac{1}{2}i
\]
\[ \frac{\sqrt{3} + 0.1}{i(-3i)} \]
Definition
A **quadratic equation in** \( x \) **is an equation that can be written in the standard form** \( ax^2 + bx + c = 0 \) **where** \( a, b, \) **and** \( c \) **are real numbers and** \( a \neq 0 \). (Also called a **second-degree polynomial equation in** \( x \).)
Solving a Quadratic Equation:

A. *Factoring* Use the Zero-Factor Property: If $ab = 0$, then $a = 0$ or $b = 0$ or both are 0.

Examples: Solve by factoring.

1. $9x^2 - 1 = 0$

   $$(3x + 1)(3x - 1) = 0$$

   $3x + 1 = 0$
   $\chi = -\frac{1}{3}$

   $3x - 1 = 0$
   $\chi = \frac{1}{3}$$
A. **Factoring** Use the Zero-Factor Property:
If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \) or both are 0.

Examples: Solve by factoring.

1. \( 9x^2 - 1 = 0 \)
2. \[ x^2 - 10x + 9 = 0 \]

\[(x - 1)(x - 9) = 0\]

\[x = 1 \quad x = 9\]
3. \[2x^2 = 19x + 33\]

\[19x + 33 - 2x^2 = 0\]

\[2x^2 - 19x - 33 = 0\]
B. Extracting Square Roots

If $u^2 = c$, and $c > 0$, then $u = \pm \sqrt{c}$. 
Examples: Solve by extracting square roots.

4. \[ 9x^2 = 25 \]

\[ x^2 = \frac{25}{9} \]

\[ x = \pm \frac{5}{3} \]

\[ 9x^2 - 25 = 0 \]

\[ (3x + 5)(3x - 5) = 0 \]

\[ 3x + 5 = 0 \quad 3x - 5 = 0 \]

\[ x = -\frac{5}{3} \quad x = \frac{5}{3} \]
5. \[(x - 5)^2 = 20\]
C. **Completing the Square**  

If \(ax^2 + bx + c = 0\), then

\[
x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2
\]

\[
(x + \frac{b}{2})^2 = c + \frac{b^2}{4}
\]
Examples: Solve by completing the square.

6. \[x^2 + 8x + 14 = 0\]
7. \[4x^2 - 4x - 99 = 0\]
D.  **Quadratic Formula**  If $ax^2 + bx + c = 0$, then

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Examples: Solve by using the quadratic formula.

8.  $x^2 - 10x + 22 = 0$
9. \[ 9x^2 + 24x + 16 = 0 \]
10. \[9x^2 - 6x - 35 = 0\]
Definition:
For the quadratic equation $ax^2 + bx + c = 0$, the quantity $b^2 - 4ac$ is called the discriminant.

If $b^2 - 4ac > 0$, there are two different real zeros.
If $b^2 - 4ac = 0$, there is one repeated real zero.
If $b^2 - 4ac < 0$, there are two complex conjugate zeros.
Solving a Quadratic Equation

**Factoring:** If \( ab = 0 \), then \( a = 0 \) or \( b = 0 \).  

**Example:** \( x^2 - x - 6 = 0 \)

\[(x - 3)(x + 2) = 0\]

\[x - 3 = 0 \quad \Rightarrow \quad x = 3\]

\[x + 2 = 0 \quad \Rightarrow \quad x = -2\]

**Extracting Square Roots:** If \( u^2 = c \), where \( c > 0 \), then \( u = \pm \sqrt{c} \).

**Example:** \((x + 3)^2 = 16\)

\[x + 3 = \pm 4\]

\[x = -3 \pm 4\]

\[x = 1 \quad \text{or} \quad x = -7\]

**Completing the Square:** If \( x^2 + bx = c \), then

\[x^2 + bx + \left(\frac{b}{2}\right)^2 = c + \left(\frac{b}{2}\right)^2\]

\[\left(x + \frac{b}{2}\right)^2 = c + \frac{b^2}{4}\].
Example: (From page 208.)

131. The distance $d$ (in miles) a car can travel on one tank of fuel is approximated by $d = -0.024s^2 + 1.455s + 431.5; 0 < s \leq 75$, where $s$ is the average speed of the car in miles per hour.
b. Use the graph to determine the greatest distance that can be traveled on a tank of fuel. How long will the trip take?
A. Polynomial Equations of Higher Degree
(Try the methods used to solve quadratic equations)

**Example** Solve.

\[ x^3 = 9x \]

\[ x^3 - 9x = 0 \]

\[ x(x^2 - 9) = 0 \]

\[ x(x + 3)(x - 3) = 0 \]

\[ x = 0 \]

\[ x + 3 = 0 \Rightarrow x = -3 \]

\[ x - 3 = 0 \Rightarrow x = 3 \]

**Tip:** Remember not to divide by \( x \) because you will lose a solution that way.
A. *Polynomial Equations of Higher Degree*
(Try the methods used to solve quadratic equations)

**Example** Solve.

\[ x^3 = 9x \]
**Tip:** Remember not to divide by $x$ because you will lose a solution that way.
\[ x^3 - x^2 - 4x + 4 = 0 \]
Examples: Solve.

1. \[20x^3 - 125x = 0\]
2. \[ x^4 + 2x^3 - 8x - 16 = 0 \]
3. \[36t^4 + 29t^2 - 7 = 0\]
4. \[ 6\left(\frac{s}{s+1}\right)^2 + 5\left(\frac{s}{s+1}\right) - 6 = 0 \]
Examples: Solve.

5. \((x^2 - x - 22)^\frac{4}{3} = 16\)
6. \[ 4x^2(x-1)^3 + 6x(x-1)^3 = 0 \]
7. \[ \sqrt{x+5} = \sqrt{x-5} \]
8. \[ \sqrt{x} + \sqrt{x-20} = 10 \]
C. Equations Involving Fractions
(Start by multiplying each side of the equations by the LCD.)

Solve.
\[
\frac{x}{x-1} - \frac{6}{x} = 2
\]
Examples: Solve.

9. \[ 4x - 1 = \frac{3}{x} \]
10. \( \frac{4}{x} - \frac{5}{3} = \frac{x}{6} \)
D. **Equations Involving Absolute Value**
(Remember to consider the quantity inside the absolute value may be positive or negative. Graph the equations to verify your results.)

Solve.

\[ |x^2 - d| = x \]
Examples: Solve.

11. $|3x+2|=7$
12. $|x-10| = x^2 - 10x$
2. Two equal rectangular lots are enclosed by fencing the perimeter of a rectangular lot and then putting a fence across its middle. If each lot is to contain 1200 square feet, what is the minimum amount of fence needed to enclose the lots? (Include the fence across the middle.)
Approximate the points of intersection of the graphs of the following equations.
\[ y = x^2 + 2x - 8 \]
\[ y = x^3 + x^2 - 6x + 2 \]

Point of intersection is \((-3.31863, -3.62396)\).
Example:

Verify the given zeros both algebraically and graphically.

\[ f(x) = x - 3 - \frac{10}{x} \quad \text{zeros } x = -2, 5 \]
7. \[
\frac{6}{x} + \frac{8}{x + 5} = 3
\]
8. \[ \sqrt{x - 4} = 8 \]
Examples: Use a graphing utility to find any points of intersection.

\[ y = \frac{1}{3} x + 2 \]

9.
\[ y = \frac{5}{2} x - 11 \]
\[ y = -x \]

10. \[ y = 2x - x^2 \]