2.1 Using Mathematical Models to Solve Equations
2.2 Solving Equations Graphically, Intercepts, Zeros
Intersections of two graphs

Worksheet #23

**Homework:** 2.2
Example 7. The average daily temperature in Dublin, Ireland in December is 4.4 degrees Celsius. What is the average daily temperature in degrees Fahrenheit? Round to the nearest degree.

\[ F = \frac{9}{5}C + 32 \]

\[ \frac{5}{9} (F - 32) = \frac{5}{9} \left( \frac{9}{5} \right) C \]

\[ F = \frac{9}{5} (4.4) + 32 \]

\[ C = \frac{5}{9} (F - 32) \]

\[ = 39.9 \]

\[ \approx 40^\circ F \]
Examples:  (from pp. 169 – 170)

Example 2. A shirt that normally sells for $30 is marked down $6. What percent of the original price is this markdown?

\[
\frac{6}{30} = \frac{1}{5} = 0.2 \Rightarrow 20\% \]

42. A picture frame has a total perimeter of 3 meters. The height of the frame is $2/3$ times its width.
   a. Draw a picture that gives a visual representation of the problem. Identify the width as $w$ and the height as $h$.

   \[ h = \frac{2}{3} w \]

   \[ \frac{2}{3} (0.9) = 0.6 \text{ m} \]

   b. Write $h$ in terms of $w$ and write an equation for the perimeter in terms of $w$.

   \[ P = 2h + 2w \]

   \[ P = 2\left(\frac{2}{3}w\right) + 2w = \frac{4w}{3} + \frac{6w}{3} = \frac{10w}{3} \]

   c. Find the dimensions of the picture frame.

   \[ P = \frac{10w}{3} \]

   \[ 3 = \frac{10w}{3} \]

   \[ \frac{9}{10} = w \text{ m} \]
44. You are taking a course that has four tests. The first three tests are 100 points each and the fourth test is 200 points. To get an A in the course, you must have an average of at least 90% on the four tests. Your scores on the first three tests were 87, 92, and 84. What must you score on the fourth test to get an A for the course?

\[
\frac{87 + 92 + 84 + x}{5} = \frac{90}{1}
\]

\[
263 + x = 450
\]

\[
-263
\]

\[
x = 187
\]

93.5
2.2 Solving Equations Graphically

Intercepts, Zeros, Solutions
Solutions graphically
Intersections of two graphs
Definitions:

The point \((a, 0)\) is called an \textbf{x-intercept} of the graph of an equation if it is a solution point of the equation. To find the x-intercept(s), set \(y\) equal to 0 and solve the equation for \(x\).

The point \((0, b)\) is called a \textbf{y-intercept} of the graph of an equation if it is a solution point of the equation. To find the y-intercept(s), set \(x\) equal to 0 and solve the equation for \(y\).
What is possible?

Three $x$-Intercepts
One $y$-Intercept

No $x$-Intercepts
One $y$-Intercept

No Intercepts

One $x$-Intercept
Two $y$-Intercepts
Example: Find the x- and y-intercepts of the graph of each equation.

\[ 2x + 3y = 6 \]

**x-intercept:** Let \( y = 0 \)

\[ 2x = 6 \]
\[ x = 3 \]

**y-intercept:** Let \( x = 0 \)

\[ 3y = 6 \]
\[ y = 2 \]

\( (3, 0) \) \hspace{1cm} \( (0, 2) \)
Find the x-intercepts and y-intercepts.

\[ y = 9x^2 + x - 6 \]

**x-intercepts:**
Let \( y = 0 \)

\[ 0 = x^2 + x - 6 \]
\[ 0 = (x + 3)(x - 2) \]
\[ x + 3 = 0 \quad \text{or} \quad x - 2 = 0 \]
\[ x = -3 \quad x = 2 \]

\((2,0), (-3,0)\)

**y-intercept:**
Let \( x = 0 \)

\[ y = 0 + 0 - 6 = -6 \]

\((0, -6)\)
Find the x-intercepts and y-intercepts.

1. \[ y = \frac{4}{3}x + 2 \]

Point at (0,2) and (1.5,0).

0 = \frac{4}{3}x + 2

\[ \frac{3}{4} - 2 = \frac{3}{4} \frac{4}{3}x \]

\[ -\frac{3}{2} = x \]
Find the x-intercepts and y-intercepts.

\[ y = 10 + 2(x - 2) \]
Find the x-intercepts and y-intercepts.

3. \( y = 3 - \frac{1}{2}|x + 1| \)

\[ y = 3 - \frac{1}{2} \]

\[ \frac{y}{2} = 2.5 \]

\[ y = 3 - \frac{1}{2} |x + 1| \]

X-intercepts: Let \( y = 0 \)

\( (0, 2.5) \)

\( (5, 0) \)

\( (-7, 0) \)

\[ 0 = 3 - \frac{1}{2} |x + 1| \]

\[ \frac{1}{2} |x + 1| = 3 \]

\[ |x + 1| = 6 \]

\[ -(x + 1) = 6 \]

\[ -x - 1 = 6 \]

\[ -x = 7 \]

\[ x = -7 \]

\[ x + 1 = 6 \]

\[ x = 5 \]
1. Using 120 feet of fencing, a farmer wishes to contain a cow in a rectangular plot of land that has one side along the barn. If no fencing is required along the barn side, what should be the dimensions of the rectangular field to provide the cow with maximum grazing area?

\[
A = 120x - 2x^2
\]

\[
x = \frac{-b}{2a}
\]
\[ A = 120x - 2x^2 \]
\[ A = -2x^2 + 120x \]

\[ x = -\frac{b}{2a} = -\frac{120}{2(-2)} = 30 \]

\[ \begin{array}{c|c|c}
\hline
y & x & 60 \\
\hline
50 & 18 & 30 \\
\hline
\end{array} \]

\[ a = -2 \]
\[ b = 120 \]
\[ c = 0 \]
\[ A = w (120 - 3w) = 120w - 3w^2 \]
2. Two equal rectangular lots are enclosed by fencing the perimeter of a rectangular lot and then putting a fence across its middle. If each lot is to contain 1200 square feet, what is the minimum amount of fence needed to enclose the lots? (Include the fence across the middle.)
3. For a charter boat to operate, a minimum of 75 people paying $125 each is necessary. For each person in excess of 75, the fare is reduced by $1 per person. Find the number of people that will make the revenue a maximum. What is the maximum revenue?

\[
\text{Revenue} = \text{Selling Price} \times \text{Number Sold}
\]

\[
R = (125 - x)(75 + x)
\]

\[
R = 9375 + 50x - x^2
\]
4. Postal restrictions limit the size of packages sent through the mail. If the restrictions are that the length plus the girth may not exceed 108 inches, find the volume of the largest box with square cross section that can be mailed.

\[ 4x + l = 108 \]
\[ l = 108 - 4x \]

\[ V = l \cdot w \cdot h \]
\[ V = (108 - 4x) \cdot x \cdot x \]
\[ V = 108x^2 - 4x^3 \]

\[ 108 - 4(18) = 36 \]
\[ 108 - 72 = 36 \]

4 \times 4 \times 36
5. A rectangular box with a square base is to be formed from a square piece of metal with 12-inch sides. A square piece with side $x$ is cut from the corners of the metal and the sides are folded up to form an open box. What value of $x$ will maximize the volume of the box?
Definition
A zero of a function is a number $a$ such that.
The following statements are equivalent:
1. The point $(a, 0)$ is an $x$-intercept of the graph of $f$.
2. The number $a$ is a zero of the function $f$.
3. The number $a$ is a solution of the equation $f(x) = 0$. 
Verify that the real numbers -5 and 1 are zeros of the function $f(x) = x^2 + 4x - 5$.

<table>
<thead>
<tr>
<th>Algebraic Solution</th>
<th>Graphical Solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f(-5) = (-5)^2 + 4(-5) - 5$</td>
<td><img src="image" alt="Graph" /></td>
</tr>
<tr>
<td>$= 25 + (-20) - 5$</td>
<td></td>
</tr>
<tr>
<td>$= 0$</td>
<td></td>
</tr>
<tr>
<td>$f(1) = 1^2 + 4(1) - 5$</td>
<td></td>
</tr>
<tr>
<td>$= 1 + 4 - 5$</td>
<td></td>
</tr>
<tr>
<td>$= 0$</td>
<td></td>
</tr>
</tbody>
</table>
Approximate the points of intersection of the graphs of the following equations.

\[ y = x^2 + 2x - 8 \]
\[ y = x^3 + x^2 - 6x + 2 \]

Point of intersection is (-3.31863, -3.62396).
Example:

Verify the given zeros both algebraically and graphically.

\[ f(x) = x - 3 - \frac{10}{x} \quad \text{zeros } x = -2, 5 \]
5. \[ 3.5x - 8 = 0.5x \] Get equation to equal zero: \[ 3.0x - 8 = 0 \]
6. \[0.60x + 0.40(100 - x) = 50\]
7. \[ \frac{6}{x} + \frac{8}{x+5} = 3 \]
8. \[ \sqrt{x - 4} = 8 \]
Examples: Use a graphing utility to find any points of intersection.

\[ y = \frac{1}{3}x + 2 \]

9. \[ y = \frac{5}{2}x - 11 \]
\[ y = -x \]

10. \[ y = 2x - x^2 \]