Mat 161 Day 7 Agenda

Business Problem
Short Review of Translation of Functions
Chapter 1.5 and P.5

Review Sheet 1.3 and P.3
Worksheet 3b on DQ

Distance Formula
Equation of Circle

Quiz
Did you remember your Code of ethics?
The cost per unit to produce a radio model is $60. The manufacturer charges $90 per unit for orders of 100 or less. To encourage large orders, the manufacturer reduces the charge by $.15 per radio for each unit ordered in excess of 100.

\[
P(x) = \begin{cases} 
30x & x \leq 100 \\
45x - 0.15x^2 & x > 100 
\end{cases}
\]

<table>
<thead>
<tr>
<th>Units</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>110</td>
<td>3135</td>
</tr>
<tr>
<td>120</td>
<td>3240</td>
</tr>
<tr>
<td>130</td>
<td>3315</td>
</tr>
<tr>
<td>140</td>
<td>3360</td>
</tr>
<tr>
<td>150</td>
<td>3375</td>
</tr>
<tr>
<td>160</td>
<td>3360</td>
</tr>
<tr>
<td>170</td>
<td>3315</td>
</tr>
</tbody>
</table>

\[
(90 - 0.15(x - 100)) - 60 \\
(90 - 0.15x + 15) - 60 \\
105 - 0.15x - 60 \\
(45 - 0.15x) x - 45x - 0.15x^2
\]
Reflections in the Coordinate Axes:
Reflections in the coordinate axes of the graph of \( y = f(x) \) are represented as follows:
1. Reflection in the \( x \)-axis: \( h(x) = -f(x) \)
2. Reflection in the \( y \)-axis: \( h(x) = f(-x) \)
Examples: Sketch the graphs of the following functions starting with the graph of one of the six common functions.

\[ h(x) = -(x - 2)^2 + 4 \]

\[ w(x) = -x + 3 \]
Example: Use the graph of \( f(x) = x^2 \) to write formulas for the functions \( g \) and \( h \) shown in the given graph.

\[
g(x) = \quad h(x) =
\]
Quick Review

Review Questions (1.3 & P.3):

1. Determine whether the equation represents $y$ as a function of $x$:

\[ x^2 + y^2 = 4 \]
\[ 2x + 3y = 4 \]

\[ y^2 = 4 - x^2 \]
\[ y = \pm \sqrt{4 - x^2} \]
$2x + 3y = 4$

$3y = 4 - 2x$

$y = \frac{4 - 2x}{3}$

$y = \left( -\frac{2}{3} \right) x + \frac{4}{3}$
2. Let \( f(x) = \begin{cases} x^2 + 2 & x \leq 1 \\ 2x^2 + 2 & x > 1 \end{cases} \) find \( f(-2) \), then \( f(1) \)

\[
f(-2) = (-2)^2 + 2 = 6
\]

\[
f(1) = 1^2 + 2 = 3
\]

\[
f(4) = 2(4)^2 + 2 = 34
\]
3. Find the domain of \( k(x) = \frac{7}{x + 4} \)

\[ x + 4 \neq 0 \]

\[ x \neq -4 \]

\[ (-\infty, -4) \cup (-4, \infty) \]
4. Factor: \((x + 3)^2 - 9y^2\)

\[\begin{align*}
(x + 3)^2 - (3y)^2 & = A^2 - B^2 \\
& = (A - B)(A + B) \\
& = [(x + 3) - 3y][(x + 3) + 3y] \\
& = (x + 3 - 3y)(x + 3 + 3y) \\
& = (x^2 + 6x + 9 - 9y^2)
\end{align*}\]
5. Reduce: \( \frac{-x-7}{21+3x} \)

\[
\frac{-x-7}{3(x+7)} = \frac{-1(x+7)}{3(x+7)} = -\frac{1}{3}
\]

\( x \neq -7 \)
If \( f(x) = x^2 - 12x \)

Find \( \frac{f(x+h) - f(x)}{h} \)

\[
\begin{align*}
  f(x+h) &= (x+h)^2 - 12(x+h) \\
  f(x+h) &= x^2 + 2xh + h^2 - 12x - 12h \\
  -f(x) &= -x^2 + 12x \\
  f(x+h) - f(x) &= 2xh + h^2 - 12h \\
  \frac{f(x+h) - f(x)}{h} &= \frac{h(2x + h - 12)}{h} \\
  &= 2x + h - 12 \quad h \neq 0
\end{align*}
\]
\[ f(x) = x^2 + x - 1, \text{ find } \frac{f(x+h) - f(x)}{h} \]
Page 55 #22. The table shows the number $y$ of Wal-Mart stores for each year $x$ from 1994 through 2001. Sketch a scatter plot of the data.

<table>
<thead>
<tr>
<th>Year, $x$</th>
<th>Number of stores, $y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1994</td>
<td>2759</td>
</tr>
<tr>
<td>1995</td>
<td>2943</td>
</tr>
<tr>
<td>1996</td>
<td>3054</td>
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<td>1997</td>
<td>3406</td>
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<td>1998</td>
<td>3599</td>
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<td>1999</td>
<td>3985</td>
</tr>
<tr>
<td>2000</td>
<td>4189</td>
</tr>
<tr>
<td>2001</td>
<td>4414</td>
</tr>
</tbody>
</table>
Distance Formula, Midpoint, and Equation of a Circle

Given two points in the coordinate plane: \((x_1, y_1), (x_2, y_2)\)

The distance \(d\) between the two points is:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.
\]
Find the distance between (2, -5) and (8, 3).

\[ d = \sqrt{(8 - 2)^2 + (3 - (-5))^2} \]
\[ = \sqrt{6^2 + 8^2} \]
\[ = \sqrt{36 + 64} \]
\[ = \sqrt{100} \]
\[ = 10 \]
The **midpoint** of the line segment joining the two points is
\[
\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right).
\]

Find the midpoint of the line segment joining the points
(-9, 5) and (4, 2).

\[
\left( \frac{-9 + 4}{2}, \frac{5 + 2}{2} \right) = \left( -\frac{5}{2}, \frac{7}{2} \right)
\]
Find the standard form of the equation of the circle with center at (2, -5) and radius 4.

\[(x - 2)^2 + (y + 5)^2 = 4^2\]

or

\[(x - 2)^2 + (y + 5)^2 = 16\]
Examples: Factor completely.

\[ x^2 - x - 6 \]

\[ 2x^2 - 3x - 14 \]

\[ 6x^2 - 29x + 35 \]

To see a review of quadratics factoring, go to the following site:

http://faculty.mc3.edu/rhofman/Flash03/Dec17/menufactor.html
Example: Use a graphing utility to find any relative minimum or relative maximum values of the function and those intervals where \( f \) is increasing and those intervals where \( f \) is decreasing.

1. \( f(x) = (x - 1)^2 (x + 2) \)

2. \( g(x) = x^3 - 6x^2 + 15 \)

3. \( h(x) = x\sqrt{4 - x} \)
Piecwise defined functions

Drawing the whole graph for one part of the function on the board, then erase the part that you do not want.

Example Sketch the graph of \( f(x) = \begin{cases} x + 1, & \text{if } x \leq 2 \\ 1 - x, & \text{if } x > 2 \end{cases} \)

Draw, then erase, the light parts.
Example: Find the coordinates of a second point on the graph of a function \( f \) if the given point is on the graph and the function is (a.) even and (b.) odd.

Given point: \((-3, 7)\)
P.4 Rational Expressions

Definitions:
The set of real numbers for which an expression is defined is its domain. Two expressions are equivalent if they yield the same value for all numbers in their domain.

The quotient of two algebraic expressions is a fractional expression.
The quotient of two polynomials is a rational expression.
Domains:
For a polynomial, the domain is all real numbers.
For expressions that contain radicals with even indices, the radicand must be greater than or equal to zero.
For rational expressions, the denominator cannot be equal to zero.

Examples:

$5x^3 - 2x^2 + 6x - 8$ is a polynomial, the domain is all real numbers.

$\sqrt{x-2}$, $x - 2 \geq 0$; the domain is $[2, \infty)$.

$\frac{x}{x-1}$, $x - 1 \neq 0$, the domain is $(-\infty, 1) \cup (1, \infty)$. 
Find the domain:

\[ 2x^2 - 5x - 2 \]

\[ 6x^2 - 9, x > 0 \]

\[ \frac{x + 1}{2x + 1} \]

\[ \sqrt{6 - x} \]
Simplifying Rational Expressions

The key to simplifying rational expressions is that the numerator and denominator should not have any common factors. If the numerator and denominator do have common factors, they can be eliminated by using

\[ \frac{ac}{bc} = \frac{a}{b} \cdot \frac{c}{c} = \frac{a}{b} \cdot 1 = \frac{a}{b} \]

Note that \( c/c = 1 \) only if \( c \neq 0 \).
Reduce:

\[
\frac{x^2 + 8x - 20}{x^2 + 11x + 10}
\]

\[
\frac{x^2 - 9}{x^3 + x^2 - 9x - 9}
\]
Simplify: \(\frac{x^3 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1}\)
\[
\frac{x^2 + 1}{x^2 - 6x + 9} \cdot \frac{x^2 - 9}{x^2 - x + 1} = \frac{(x + 1)(x^2 - x + 1)}{(x - 3)^2} \cdot \frac{(x + 3)(x - 3)}{x^2 - x + 1} \\
= \frac{(x + 1)(x^2 - x + 1)(x + 3)(x - 3)}{(x - 3)^2 (x^2 - x + 1)} \\
= \frac{(x + 1)(x + 3)}{(x - 3)} \cdot \frac{|x^2 - x + 1|(x - 3)}{|x^2 - x + 1|(x - 3)} \\
= \frac{(x + 1)(x + 3)}{(x - 3)}, x \neq 3, 1, 3
\]
Simplify: \[ \frac{x+6}{x} + \frac{2}{x^2 + x} \]
\[
\frac{x + 6}{x} + \frac{2}{x^2 + x} = \frac{x + 6}{x} \cdot \frac{x + 1}{x + 1} + \frac{2}{x(x + 1)} \\
= \frac{(x^2 + 7x + 6) + 2}{x(x + 1)} \\
= \frac{x^2 + 7x + 8}{x(x + 1)}
\]
Complex Fractions:
Fractions that contain fractions in the numerator or denominator are called complex fractions. There are two ways to simplify a complex fraction. Here are examples of each.
Order of Operations:

\[
1 - \frac{2}{x} \cdot \frac{x}{5} + \frac{4}{x} = \frac{x - 2}{x} \cdot \frac{x}{9x - 4}
\]

\[
= \frac{x - 2}{x} \cdot \frac{x(x - 1)}{9x - 4}
\]

\[
= \frac{(x - 2)(x - 1)}{9x - 4}, \quad x \neq 0
\]
Multiply by the LCD.

\[
\begin{align*}
\frac{1 - \frac{2}{x}}{\frac{5}{x - 1} + \frac{4}{x}} &= \frac{1 - \frac{2}{x}}{\frac{5}{x - 1} + \frac{4}{x}} \cdot \frac{x(x - 1)}{x(x - 1)} \\
&= \frac{x(x - 1) - 2(x - 1)}{5x + 4(x - 1)} \\
&= \frac{x^2 - 3x + 2}{9x - 4}
\end{align*}
\]
Rationalize the numerator: \[
\frac{\sqrt{x} + 1 - 1}{x}
\]
\[
\frac{\sqrt{x+1} - 1}{x} = \frac{\sqrt{x+1} - 1}{x} \cdot \frac{\sqrt{x+1} + 1}{\sqrt{x+1} + 1} \\
= \frac{\sqrt{x+1}^2 - 1^2}{x(\sqrt{x+1} + 1)} \\
= \frac{x}{x(\sqrt{x+1} + 1)} \\
= \frac{1}{\sqrt{x+1} + 1}, x \neq 0
\]
Perform the operations and simplify:

\[
\frac{4y - 16}{5y + 15} \cdot \frac{2y + 6}{4 - y}
\]
\[
\frac{2}{x^2 - x - 2} + \frac{10}{x^2 + 2x - 8}
\]
\[ \frac{2}{x+1} + \frac{2}{x-1} + \frac{1}{x^2-1} \]
\[ \frac{x - 4}{x - \frac{4}{4}} = \frac{x}{x} \]
\[ 5x^5 - 3x^{\frac{-3}{2}} \]
\[2x(x - 5)^{-3} - 4x^2(x - 5)^{-4}\]