Chapter 2, Section 4 - Curve Fitting and Linear Regression

1. Curve Fitting

There are three forms of the equation of a line which we are interested in. You already know that if the equation is \( y = b \), the graph is a horizontal line, and if the equation is \( x = b \) the graph will be a vertical line. If a line is not horizontal or vertical, then its equation has one of the following forms:

- **Slope-intercept form:** \( y = mx + b \)
- **Standard form:** \( Ax + By = C \)
- **Point-slope form:** \( y - y_1 = m(x - x_1) \)

**Example:**

Find the equation of a line whose slope is \(-\frac{3}{2}\) and passes through the point \((3, 4)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - 4 = -\frac{3}{2}(x - 3)
\]

\[
y = -\frac{3}{2}x + \frac{9}{2}
\]

\[
y = -\frac{3}{2}x + \frac{9}{2}
\]

\[
x + \frac{3}{2}y = \frac{11}{2}
\]

**Standard form:**

\[
2x + 3y = 11
\]

**Example:**

Given slope of \( f \) and \((2, -1)\), find the equation.

\[
y - 1 = -\frac{3}{2}(x - 2)
\]

\[
y = -\frac{3}{2}x + 1
\]

**Standard form:**

\[
2x + 3y = 1\]

**Example:**

Medical Insurance

According to the U.S. Bureau of the Census, the number of people living in the United States with no health insurance grew from 33,000,000 in 1990 to 37,000,000 in 1994. (Source: Statistical Abstract of the United States, 1998.) Assuming constant growth since 1991, how many people living in the United States in 2002 will lack health insurance?

1991: 33,000,000

1994: 37,000,000

\[
\text{M} = \frac{37,000,000 - 33,000,000}{4} = 1,000,000
\]

\[
y - 33,000,000 = 1,000,000(x - 1991)
\]

\[
y = 33,000,000 + 1,000,000(x - 1991)
\]

\[
y = 33,000,000 + 1,000,000(2002 - 1991)
\]

\[
y = 33,000,000 + 1,000,000(11)
\]

\[
y = 33,000,000 + 11,000,000
\]

\[
y = 44,000,000
\]

11.78 million people

\[
1995: 37,000,000
\]

\[
y = 33,000,000 + 1,000,000(x - 1991)
\]

\[
y = 33,000,000 + 1,000,000(1995 - 1991)
\]

\[
y = 33,000,000 + 1,000,000(4)
\]

\[
y = 33,000,000 + 4,000,000
\]

\[
y = 37,000,000
\]

2.6 million people
II. Linear Regression

Let suppose you have many points, not just two, to consider. Consider the following problem.

SHOPPING CENTERS. The number of shopping centers in the United States has grown in recent years, as shown in the following table. Use linear regression to fit a linear function to the data, and graph the line and the data.

<table>
<thead>
<tr>
<th>Year</th>
<th>Number of Centers</th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>7,600</td>
</tr>
<tr>
<td>1972</td>
<td>13,174</td>
</tr>
<tr>
<td>1976</td>
<td>17,523</td>
</tr>
<tr>
<td>1980</td>
<td>22,650</td>
</tr>
<tr>
<td>1984</td>
<td>25,508</td>
</tr>
<tr>
<td>1988</td>
<td>32,563</td>
</tr>
<tr>
<td>1992</td>
<td>38,966</td>
</tr>
<tr>
<td>1996</td>
<td>42,130</td>
</tr>
</tbody>
</table>


Let \( x \) = the number of years since 1964 and \( f \) = the number of shopping centers, in thousands. Plot those points and then find an equation that “best” fits through the points. Use the equation to predict the number of shopping centers in 1990, in 2002.

Plot the points on your calculator. If all the points fall on the same straight line, we just have to pick any two to write the equation.

Statisticians have found that the line that fits best is the regression line whose slope and y-intercept have the following equations.

\[
\begin{align*}
    m &= \frac{n \sum (x_i y_i) - \sum x_i \sum y_i}{n \sum (x_i)^2 - \left( \sum x_i \right)^2} \\
    b &= \frac{\sum y_i - m \sum x_i}{n}
\end{align*}
\]

But, your calculator knows these formulas...so...