Sections 6 and 7: Difference of Squares and Sum and Difference of Cubes

Factoring techniques so far:

I. Factoring Technique #1: Factor Out Common Factors
II. Factoring Technique #2: Factor by Grouping
III. Factoring Technique #3: Factor Trinomials of the Form $ax^2 + bx + c$, $a \neq 1$

IV. Factoring Technique #4: Factor Trinomials of the Form $ax^2 + bx + c$, $a = 1$

Two new techniques that we need to look at are:

Factor the Difference of Perfect Squares and Factor the Sum and Difference of Perfect Cubes

V. Factoring Technique #5: Factor the Difference of Perfect Squares

\[ a^2 - b^2 \]

Example: \[ x^2 - y^2 = (x + y)(x - y) \]

Factor completely:

a.) \[ x^2 - 9 = (x + 3)(x - 3) \]

b.) \[ 25y^4 - 49x^2 = (5y^2 + 7x)(5y^2 - 7x) \]

c.) \[ x^4 - y^4 = (x + y)(x^2 + y^2)(x + y)(x - y) \]

d.) \[ 16x^4 - 81y^2 = 5(1 - x^2)(1 + x^2)(1 + 9y^2) \]

VI. Factoring Technique #6: Factor the Sum and Difference of Cubes

Example: \[ x^3 + y^3 = \]

\[ x^3 - y^3 = \]

Factor completely:

a.) \[ t^3 - 8 = \]

b.) \[ x^3 + 27 = \]

Solve: \[ x^3 + 3x^2 - 4x - 12 = 0 \]

Solve: \[ x^4 - 8x^2 + 16 = 0 \]
Section 8. Applications

Example 1: Fireworks Displays. Fireworks are typically launched from a mortar with an upward velocity (initial speed) of about 64 ft/sec. The height \( h(t) \), in feet, of a "weeping willow" display, \( t \) seconds after having been launched from an 80-ft. high rooftop, is given by:

\[ h(t) = -16t^2 + 64t + 80. \]

After how long will the cardboard shell from the fireworks reach the ground?

Example 2. Carpentry. In order to build a deck at a right angle to their house, Lucinda and Felipe decide to plant a stake in the ground a precise distance from the back wall of their house. This stake will combine with two marks on the house to form a right triangle. From a course in geometry, Lucinda remembers that there are three consecutive integers that can work as sides of a right triangle. Find the measurements of that triangle.