Section 3 - Polynomial Equations and Factoring

Why factor?
Consider the following problem. Suppose that a baseball is thrown upward. Its height, \( h(t) \) in feet, after \( t \) seconds is given by
\[
h(t) = -16t^2 + 68t.
\]

**Note:**
What does this mean?

If \( t = 0 \), \( h = \)

If \( t = 1 \), \( h = \)

If \( t = 2 \), \( h = \)

If \( t = 3 \), \( h = \)

If \( t = 4 \), \( h = \)

If \( t = 5 \), \( h = \)

Thus, the ball hits the ground after __ seconds.

We need an algebraic method to answer the question of when the ball hits the ground. In fact, we need to know how to factor.

We just solved the equation \(-16t^2 + 68t = 0\). Now graph the function \( y = -16x^2 + 68x \). Use the window: \((-5, 5, 1, -20, 80, 10)\).

What is the degree of \( y = -16x^2 + 68x \)? This graph could cross the \( x\)-axis times.

The values of \( x \) which make \( y \) or \( f(x) = 0 \) are called zeros.

The zeros of the function are and .

List of factoring techniques

1. Factoring Technique #1: Factor out common factors

**Example:** \( 3x^3 - 8 = 4(x^3 - 2) \)

Factor completely:

\[
\begin{align*}
\text{a.) } 10p^3q - 4p^3q - 2p^3q^3 &= \\
\text{b.) } 4x - 24 &= \\
&= \frac{4}{8}(5p - 2p - 2) - 4(x + 6) \\
&= 2p + p - 2 - 2 + 8 - 2x - 6 \\
&= -2x - 2 + 2 \\
&= -2x + 0
\end{align*}
\]
c) $-x^2 + 5x$

d) $(a - b)x + 5 + (a - b)x - y^2$

$$\begin{align*}
  & \frac{-(a-b)(x+5+y^2)}{(a-b)(ax+5-y^2)} \\
  & \frac{(x^2+7y+2)}{4x+7} \\
  & (x+7)(y+2-4) \\
  & (x+7)(y-2)
\end{align*}$$

II. Factoring Technique #2: Factor by Grouping

Example: $t^2 + 3t^2 + 4t + 12$

Factor completely:

$$\begin{align*}
  & t(t+3) + 4(t+3) \\
  & t(3)(2t+4) \\
  & t^2(t+4) - 2(t+6) \\
  & 4x^2 + 20x - 15 - 3x \\
  & 4x^2 + 5x - 3(5x + x) \\
  & (x+5)(4x^2 - 3)
\end{align*}$$

Solve:

a) $-16t^2 + 64t = 0$

$$\begin{align*}
  & -16t(t-4) = 0 \\
  & t-4 = 0 \\
  & t = 4
\end{align*}$$

b) $2x - 4x^2 = 0$

$$\begin{align*}
  & 2x(1-2x) = 0 \\
  & 2x = 0 \\
  & 1-2x = 0 \\
  & x = 0 \\
  & \frac{1}{2} = \frac{2x}{2} \\
  & \frac{1}{2} = x
\end{align*}$$