MAT223 Exam Review

Determine if a DE is linear or nonlinear and how many arbitrary constants are in the general solution. (#1)

Sketch the phase portrait for a DE of the form \( \frac{dx}{dt} = x(a - bx) \). (#1)

Identify the trajectory of a periodic solution. (pp. 144 - 146)

Verify a particular solution to a 2\textsuperscript{nd} order linear DE. (#1)

Find the general solution to a 2\textsuperscript{nd} order homogeneous linear DE. (#2)

Find the particular solution to a 2\textsuperscript{nd} order linear DE. (#2)
Given the graph of \( f(t) \), find the graph of a function involving 
\( f(t - \alpha) \) and/or \( U(t - \alpha) \).

Eliminate one variable from a system of linear first-order DE's.

Evaluate 3 Laplace transforms.

Solve 2 DE's by separation of variables or a method of your choice.

Define local truncation error.

Use a Wronskian to determine whether or not a set of functions is linearly independent.
Sketch a trajectory for a 2nd order linear DE. (#2)
For this same DE, classify (0, 0) as unstable, stable (not asymptotically stable), or asymptotically stable. (#2)
Find the characteristic equation for a system of DE’s. (#3)
For the same system, find the eigenvalues and eigenvectors of the matrix of coefficients. (#3)
Given the DE and formulas for \( W, W_1, W_2 \), use the method of variation of parameters to find solution. (#2)
Set up the DE for an a spring problem. (#2)

Given $y_c$ for a linear DE, what functions should be tried for $y_p$ using the method of undetermined coefficients. (#2)

Describe the difference between Euler and Runge-Kutta. (Section 2.5) (Errors)
Find the recurrence formula for the power series solution to a DE.

Given the recurrence formula, find a few coefficients.

Find the singular points of a DE.

Determine whether a given singular point is regular or not.

Find the indicial roots for a Frobenius-type solution.

\[ y' - 2y = 0 \]

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<thead>
<tr>
<th>( c_0 )</th>
<th>( c_1 )</th>
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<tbody>
<tr>
<td>( c_2 = 3c_1 )</td>
<td>( c_3 = c_1 )</td>
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<tr>
<td>( c_4 = \frac{1}{3}c_2 = \frac{1}{3}(3c_0) = c_0 ) [ (k = 2) ]</td>
<td>( c_5 = -\left(\frac{3-3}{3+1}\right)c_2 = 0 ) [ (k = 3) ]</td>
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<td>( c_6 = -\frac{1}{5}c_4 = -\frac{1}{5}(c_3) = -\frac{1}{5}c_1 )</td>
<td>( c_7 = \left(\frac{5-3}{5+1}\right)c_2 = 0 )</td>
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Examples: Determine the singular points and classify them as regular or irregular.

1. \( y'' - y' + \frac{1}{x-1}y = 0 \)

0 is a regular singular point
1 is an irregular singular point

\( \square \) and \( \square \)
Remember, $c_n$ cannot be 0

$[2k(k-1)-2(k+1)]c_k x^k = 0 \quad c_k \neq 0$  $x^r$ is not identically 0

$2(r-1)r - r + 1 = 0$  (indicial equation)

$2r^2 - 2r - r + 1 = 0$

$2r^2 - 3r + 1 = 0$  $(2r-1)(r-1) = 0$

$r = \frac{1}{2}, 1$  indicicial roots