MAT 106
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6.1 Order of Operations

**Algebra** – from the Arabic word *al-jabr* meaning “reunion of broken parts.”

**Variables** – letters used to represent numbers

\[ x, y, z \]

**Constants** – symbols used to represent specific quantities

\[ 2, 5, -7, \pi \]

**Algebraic expression** – collection of variables, numbers, parentheses, and operation symbols.

**Example:**

\[
\frac{x^2 - 5xy + \sqrt{y}}{x(3-x)}
\]
To evaluate an expression is to find the value of the expression for a given value of the variable.

Order of Operations

1. Perform all operations within parentheses or other grouping symbols.

2. Perform all exponential operations (raising to powers or taking of roots).

3. Perform all multiplications and divisions from left to right.

4. Perform all additions and subtractions from left to right.

(Please Excuse My Dear Aunt Sally.)
Examples: Evaluate the expression for the given value or values of the variable.

1. \(x^2, x = -8\) 
   \((-8)^2 = 64\)

2. \(-x^2, x = -4\) 
   \((-4)^2 = 16\)
   
   \(-x^2 \neq (-x)^2\)

3. \(5x^2 + 7x - 11, x = -1\) 
   
   \(5(-1)^2 + 7(-1) - 11\) 
   \(= 5(1) + 7(-1) - 11\) 
   \(= 5 - 7 - 11 = -13\)
4. \(-x^2 + 4xy, x = 2, y = 3\)

\[-2^2 + 4(2)(3) = -4 + 24\]
\[-(2^2) = 20\]

5. \((x + 3y)^2, x = 4, y = -3\)

\[(4 + 3(-3))^2 = (4 - 9)^2 \]
\[= (-5)^2 = 25\]
Two algebraic expressions joined by an equal sign form an equation.

The solution to an equation is the number or numbers that replace the variable to make an equation true.

Examples: Determine whether the value(s) is (are) a solution to the equation.

1. \[2x^2 - x - 5 = 0, \ x = 3\]

\[2(3)^2 - 3 - 5 = \]

\[2(3) - 3 - 5 = 10 \neq 0\]

\[\text{no}\]
2. \[5x - 7 = -27, x = -4\]

\[5(-4) - 7 = -20 - 7 = -27\]

\(\text{yes}\)

3. \[y = x^2 + 3x - 5, x = 1, y = -1\]

\[-1 \overset{?}{=} 1^2 + 3(1) - 5\]

\[-1 \overset{?}{=} 1 + 3 - 5\]

\[-1 = -1 \text{ (yes)}\]
6.2 Linear Equations in One Variable

**Terms** – parts that are added or subtracted in an expression.

**Numerical coefficients (coefficients)** – usually, the number at the beginning of a term.

(In the term $xy$, the coefficient is 1; in the term $-xy$, the coefficient is $-1$.)

**Like terms** – terms that have the same variables with the same exponents. If terms are not like, they are **unlike**.
Example: In the expression $2x^3 - 5x^2 + x - 4$, the terms are $2x^3$, $-5x^2$, $x$, and $-4$ and the coefficients are $2$, $-5$, $1$, and $-4$.

Example: Find the like terms in the following list:

$3x^2$, $-x^2y$, $5xy^2$, $-2x$, $3xy^2$, $7x^2y$.

What are the coefficients of the original terms:

$3$, $-1$, $5$, $-2$, $3$, $7$.
To **simplify** an expression means to combine like terms.

To combine like terms, use the properties on p. 293. One of these properties is the Distributive Law:
\[ a(b + c) = ab + ac \]

Example:
\[ 5(3 + 4) \stackrel{?}{=} 5(3) + 5(4) \]
\[ 5(7) = 15 + 20 \]
\[ 35 = 35 \quad \checkmark \]
Examples: Combine like terms:

1. \(-4x - 7x = \boxed{-11x}\)

2. \(x - 4x + 3 = \boxed{-3x + 3}\)

3. \(\frac{2}{3}x + \frac{1}{6}x - 5 = \)

   \(\frac{2}{3}x + \frac{1}{6}x - 5 = \boxed{\frac{5}{6}x - 5}\)
4. \[ 6(r - 3) - 2(r + 5) + 10 = \]

\[ 6r - 18 - 2r - 10 + 10 = \]

\[ 4r - 18 \]

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Homework:
6.1 (all)
6.2 (up to #23 odd)