13.7 The Normal Curve

Skip other types of distributions on pp. 796 - 797

The Normal or Gaussian distribution is a symmetric distribution often described as a bell-shaped curve.

In a normal distribution, the mean, median, and mode all have the same value.

*z*-scores or standard scores determine how far, in standard deviations, a given score is from the mean of the distribution.

\[
z = \frac{x - \bar{X}}{s} = \frac{X - \mu}{\sigma}
\]
population: mean $\mu$ standard deviation $\sigma$

Sample: mean $\bar{X}$ standard deviation $s$

mean $\mu$

median

mode

smaller standard deviation

larger standard deviation
Tests:
English
mean 85
stand. deviation 10
your score (x) 90
On which test did you do better relative to the class?
\[ z = \frac{90 - 85}{10} = 0.5 \]

Math
mean 80
stand. deviation 4
your score (x) 88
\[ z = \frac{88 - 80}{4} = 2 \] far better
The **standard normal** distribution is the normal distribution with mean = 0 and standard deviation = 1.
Examples:
Use the normal probability table to find the area (probability):
Assume a standard normal distribution.

1. Below the mean

\[ P(z < 0) = .5 \]

Total area = 1
Symmetric about mean = 0
2. Between 1.10 and 1.80 standard deviations above the mean

\[ P(1.10 \leq z \leq 1.80) = .464 - .364 = .100 \]

\[ \approx 10\% \]
3. To the right of $-1.78$

$$P(z > -1.78) = .463 + .5 = .963$$
4. To the left of 1.96

\[ P(z < 1.96) = 0.5 + 0.475 = 0.975 \]
4b. Between $-1.5$ and $1.5$.

$$P(-1.5 \leq z \leq 1.5) = 2(0.433)$$

$$= 0.866$$

Homework Section 13.7 up to #48.
### TABLE 13.7 Areas under the Standard Normal Curve (the \( z \)-table)

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The column under \( A \) gives the area under the entire curve that is between \( z = 0 \) (or the mean) and a positive value of \( z \).
13.6 Measures of Dispersion

Measures of dispersion are used to indicate the spread of data.

The **Range** = highest value - lowest value

The **Standard deviation** = \[ \sqrt{\frac{\sum (X - \bar{X})^2}{n-1}} \] = \( \sigma \)
3. For the employees salaries in the previous section:

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<th>$(X - \bar{X})^2$</th>
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Range = 79000 - 22000 = 57000

\[
\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n} \cdot \frac{341000}{10} = 34100
\]

\[
s = \sqrt{\frac{\sum_{i=1}^{n} (X_i - \bar{X})^2}{n-1}} = \sqrt{\frac{34929000000}{9}} = \sqrt{3881000000} = 19700.3
\]
31. *Height and Weight Distribution* The chart shown below uses the symbol $\sigma$ to represent the standard deviation. Note that $2\sigma$ represents the value that is two standard deviations above the mean; $-2\sigma$ represents the value that is two standard deviations below the mean. The unshaded areas, from two standard deviations below the mean to two standard deviations above the mean, are considered the normal range. For example, the average (mean) 8-year-old boy has a height of about 50 inches, but any heights between approximately 45 inches and 55 inches are considered normal for 8-year-old boys. Refer to the chart below to answer the following questions.
a) What happens to the standard deviation for weights of boys as the age of boys increases? What is the significance of this fact?
b) At age 16, what is the mean weight, in pounds, of boys?
c) What is the approximate standard deviation of boys' weights at age 16?
d) Find the mean weight and normal range for boys at age 13.
e) Find the mean height and normal range for boys at age 13.
f) Assuming that this chart was constructed so that approximately 95% of all boys are always in the normal range, determine what percentage of boys are not in the normal range.

a. increases, more variation in weights
b. 132 lb.
\[ \sigma = 22 \]
c. 176 - 132 = 44 = 20
\[ n = 100 \]
rangle: 60 - 140
e. 

f. 

Title: Apr 12 - 9:14 AM (14 of 14)