Section 12.4 (Homework)

#19a.) \[ i \quad Pi \quad Ai \]

\[
\begin{array}{ccc}
1 & \#1 \text{ off} & 7/10 \\
2 & \#2 \text{ off} & 2/10 \\
3 & \#3 \text{ off} & 1/10 \\
\end{array}
\]

\[ E = P_1 A_1 + P_2 A_2 + P_3 A_3 \]

\[ = \frac{7}{10} (1) + \frac{2}{10} (2) + \frac{1}{10} (5) \]

\[ = \$1.60 \]
Birthday Problem

Mar. 2  Oct. 22  Feb. 7
Dec. 30  Nov. 9   June 9
Apr. 27  July 1   Sept. 26
Sept. 9  Feb. 16  Sept. 7
Apr. 23  Oct. 15  Nov. 22
Sept. 4  Mar. 25  Apr. 14
12.6 OR and AND Problems
(Compound probabilities)

**OR** – requires a successful outcome for **at least one** of the given events.

**Addition Formula:**

\[ P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \]

Two events are **mutually exclusive** if it is impossible for both events to occur simultaneously.

\[ P(A \text{ or } B) = P(A) + P(B) \]
Examples:

One card is selected from a deck of cards. Find the probability of selecting

1. a jack or a diamond

\[ P(J \text{ or } D) = P(J) + P(D) - P(J \text{ and } D) \]
\[ = \frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52} = \frac{4}{13} \text{ or .308} \]

2. a heart or a black card (mutually exclusive)

\[ P(H \text{ or } B) = P(H) + P(B) \]
\[ = \frac{13}{52} + \frac{26}{52} = \frac{39}{52} = \frac{3}{4} \text{ or .75} \]

3. a card greater than 8 or a black card

<table>
<thead>
<tr>
<th>G</th>
<th>includes 9s, 10s, J, Q, K, 8, 9's, 10's, K's</th>
</tr>
</thead>
<tbody>
<tr>
<td>How many G's are there?</td>
<td>20</td>
</tr>
</tbody>
</table>

\[ P(G \text{ or } B) = P(G) + P(B) - P(G \text{ and } B) \]
\[ = \frac{20}{52} + \frac{26}{52} - \frac{10}{52} \]
\[ = \frac{36}{52} = \frac{9}{13} \text{ or .692} \]
We use the following for sequences of events:

**AND** – requires a favorable outcome in each of the given events.

**Multiplication Rule**

\[ P(A \text{ and } B) = P(A)P(B) \]

assuming that event A has occurred. (Always assume that A has occurred before calculating the probability of event B.)

Events A and B are **independent** events if the occurrence of either event in no way affects the probability of occurrence of the other. (with replacement)

If A and B are not independent, they are **dependent** events. (without replacement)
Examples:

A cooler contains 6 cans of soda: 3 colas, 2 orange, and 1 root beer. Two cans will be selected at random. Find the probability of selecting each of the following:

a.) with replacement
b.) without replacement

First, find the sample space for both with replacement and without replacement.
b) without replacement

1st
- C: 3/6
- O: 1/6
- R: 2/6

2nd
- CC: 2/5
- CO: 3/5
- CR: 1/5
- OR: 2/5
- RO: 3/5

Sample space:
- CC
- CO
- CR
- OR
- RO

\[
\begin{array}{c|c}
\text{Sample Space} & P( ) \\
\hline
\text{CC} & \frac{6}{30} = \frac{1}{5} \\
\text{CO} & \frac{6}{30} = \frac{1}{5} \\
\text{CR} & \frac{2}{30} = \frac{1}{10} \\
\text{OR} & \frac{6}{30} = \frac{1}{5} \\
\text{RO} & \frac{2}{30} = \frac{1}{15} \\
\end{array}
\]
4. no colas

\[ P(\text{no colas}) = \]

\[ a) \text{ with replacement} \quad P(\text{OO or RR or RO or RR}) \]
\[ = P(\text{OO}) + P(\text{OR}) + P(\text{RO}) + P(\text{RR}) \]
\[ = \frac{4}{36} + \frac{2}{36} + \frac{2}{36} + \frac{1}{36} = \frac{9}{36} = \frac{1}{4} \]

\[ b) \text{ without replacement} \quad P(\text{no colas}) = \]
\[ = P(\text{OO or OR or RO}) \]
\[ = P(\text{OO}) + P(\text{OR}) + P(\text{RO}) \]
\[ = \frac{1}{15} + \frac{1}{15} + \frac{1}{15} = \frac{3}{15} = \frac{1}{5} \]

\[ P(\text{at least one cola}) = 1 - P(\text{no colas}) \]

\[ a) \text{ with replacement:} \quad P(\text{at least one cola}) = 1 - \frac{1}{4} = \frac{3}{4} \]

\[ b) \text{ without replacement:} \quad P(\text{at least one cola}) = 1 - \frac{1}{5} = \frac{4}{5} \]
<table>
<thead>
<tr>
<th>Pictures on Reels</th>
<th>Reel 1</th>
<th>Reel 2</th>
<th>Reel 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cherries</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Oranges</td>
<td>5</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>Plums</td>
<td>6</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Bells</td>
<td>3</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>Melons</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Bars</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>7’s</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Totals 22 22 22

For this slot machine, find the probability of obtaining

6. oranges on all three reels
   \[ P(000) = P(0)P(0)P(0) = \left( \frac{5}{22} \right) \left( \frac{4}{22} \right) \left( \frac{5}{22} \right) \]
   \[ = \frac{25}{2662} \approx 0.00939 \]

7. three 7’s
   \[ P(777) = \left( \frac{1}{22} \right) \left( \frac{1}{22} \right) \left( \frac{1}{22} \right) = \left( \frac{1}{22} \right)^3 \]
   \[ = 0.0000939 \]
Birthday Problem

The probability that at least 2 people in any group have the same birthday.

\[ P(\text{no matches}) = \]

\[ 1 - \frac{364}{365} \cdot \frac{363}{365} \cdot \frac{362}{365} \cdot \ldots \cdot \frac{348}{365} = \frac{365}{365} = 1 - 0.653 = 0.347 \]

\[ P(\text{at least one match}) = 1 - 0.653 = 0.347 \]