12.1 The Nature of Probability

An **experiment** is a controlled operation that yields a set of results.

The possible results of an experiment are called its **outcomes**. (simple events)

An **event** is a subcollection of the outcomes of an experiment.
Empirical probability is the relative frequency of occurrence of an event and determined by actual observations of an experiment.

\[
P(E) = \frac{\text{number of times event E has occurred}}{\text{total number of times experiment was repeated}}
\]

Theoretical probability is determined through a study of possible outcomes that can occur for the given experiment.
Examples:

1. Roll a die 50 times and record the results. Find the empirical probability of rolling a
   a.) 1 \[ P(1) = \frac{12}{50} = \frac{6}{25} = 0.24 \]
   b.) 6 \[ P(6) = \frac{11}{50} = 0.22 \]
   c.) Does the probability of rolling a 1 appear to be the same as the probability of rolling a 6?

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<thead>
<tr>
<th>no. of dots</th>
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<th>frequency (count)</th>
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<tbody>
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<td>12</td>
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<td>2</td>
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<td>6</td>
<td>HHH HHH HHH HHH HHH HHH HHH HHH HHH HHH HHH</td>
<td>11</td>
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2. **Birds at a Feeder.** The last 20 birds that fed at the Zwick's bird feeder were 10 finches, 7 cardinals, and 3 blue jays. Use this information to determine the empirical probability that the next bird to feed from the feeder is a

a.) Finch
   \[ P(\text{finch}) = \frac{10}{20} = \frac{1}{2} \]

b.) Cardinal
   \[ P(\text{cardinal}) = \frac{7}{20} \]

c.) blue jay
   \[ P(\text{blue jay}) = \frac{3}{20} \]
3. Mr. Doole's grade distribution over the past 3 years for a course in college algebra is shown in the chart below.

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**Total 645**

If Sue Gilligan plans on taking college algebra with Mr. Doole, determine the empirical probability she receives a grade of:

a.) \( P(A) = \frac{43}{645} = .0667 \)

b.) \( P(C) = \frac{260}{645} = .403 = \frac{52}{129} \)

c.) a grade of D or higher

\[
P(D \text{ or higher}) = P(A, B, C, \text{ or } D) = \frac{575}{645}
\]

= .891 = \frac{115}{129}
12.2 Theoretical Probability

Equally likely outcomes: each outcome of an experiment has the same chance of occurring as any other outcome.
Theoretical probability:

\[ P(E) = \frac{\text{number of outcomes favorable to } E}{\text{total number of possible outcomes}} \]

Event \( E \) is impossible: \( P(E) = 0 \)

Event \( E \) is certain: \( P(E) = 1 \)

\( 0 \leq P(E) \leq 1 \)

The sum of the probabilities of all possible outcomes of an experiment is 1.

The opposite or complement of \( A \) is not \( A \) or \( \overline{A} \).

\[ P(A) + P(\overline{A}) = 1 \]

\[ P(A) = 1 - P(\overline{A}) \]

\[ P(\overline{A}) = 1 - P(A) \]
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If Sue Gilligan plans on taking college algebra with Mr. Doole, determine the empirical probability she receives a grade of

c. D or higher

(alternative) The complement of "D or higher" is F or I.

\[ P(F \text{ or } I) = P(\overline{D}) = \frac{70}{645} \]

\[\Rightarrow P(D) = 1 - P(\overline{D}) = 1 - \frac{70}{645} = \frac{575}{645} \approx 0.891 = \frac{115}{129} \]
An ordinary deck of cards contains:

52 cards total
13 spades; 13 hearts;
13 diamonds; 13 clubs
4 each of
A, 2, 3, 4, 5, 6, 7, 8, 9,
10, J, Q, K
Examples:

One card is selected at random from a deck of cards. Find the probability that the card selected is

1. a 4 or 5

\[ P(4 \text{ or } 5) = \frac{8}{52} = \frac{2}{13} = 0.154 \]
2. the queen of hearts
\[ P(\text{Queen of hearts}) = \frac{1}{52} = .0192 \]

3. a red card
\[ P(\text{Red}) = \frac{26}{52} = \frac{1}{2} = .5 \]

4. a red card and a black card
\[ P(\text{a single card is black and red}) = 0 \]